Math 290 Number Theory for Teachers
Homework 8
Due: Wednesday, March 26, 2014

1. Does the ISBN 10 check digit scheme catch all single-digit replacement errors? Justify why it does or give an example showing that it doesn't.

2. Does the ISBN 13 check digit scheme catch all single-digit replacement errors? Justify why it does or give an example showing that it doesn't.

3. Does the ISBN 10 check digit scheme catch all transposition (swap of adjacent numbers) errors? Justify why it does or give an example showing that it doesn't.

4. Does the ISBN 13 check digit scheme catch all transposition errors? Justify why it does or give an example showing that it doesn't.

5. Discuss briefly the advantages and disadvantages of the ISBN 10 system and the ISBN 13 system. You may want to consider: what kinds of errors the check digits will catch or not, ease of computation, the number of books each can accommodate, etc.

6. Find all x in $\mathbb{Z}$ that satisfy $x \equiv 14 \mod 65$ and $x \equiv 25 \mod 93$.

7. Can there be a number x that satisfies $x \equiv a_1 \mod m_1$ and $x \equiv a_2 \mod m_2$ if m_1 and m_2 are not relatively prime? What is the condition on a_1 and a_2 for there to be a solution?

8. Find all x in $\mathbb{Z}$ that satisfy $x \equiv 3 \mod 7$, $x \equiv 2 \mod 11$, and $x \equiv 4 \mod 13$.

9. Find all x in $\mathbb{Z}$ that satisfy $3x \equiv 5 \mod 23$, $5x \equiv 7 \mod 24$ and $7x \equiv 3 \mod 25$.

10. Find all solutions in $\mathbb{Z}_{143}$ to $x^2 \equiv 12 \mod 143$ by first solving this equation mod the factors of 143. ($143 = 11 \cdot 13$.)