

MATH 290 NUMBER THEORY FOR TEACHERS

HOMEWORK 5

DUE: FRIDAY, FEBRUARY 28, 2014

1. For the following m , decide whether U_m has a generator: $m = 4, 5, 7, 8, 11, 15, 16$.
2. Find a generator for U_{29} . Use it to make a table of logarithms.
3. Use your table of logarithms to solve $13x^3 \equiv 21 \pmod{29}$.
4. Use your table of logarithms to solve $x^4 \equiv 7 \pmod{29}$. (That is, find all fourth roots of 7 in $\mathbb{Z}_{29}$.)
5. Use your table of logarithms to solve $x^7 = 18 \pmod{29}$.
6. Find all solutions to the following congruences.
 - (a) $4x = 2 \pmod{6}$
 - (b) $6x = 2 \pmod{10}$
 - (c) $4x = 3 \pmod{10}$
7. Without actually finding the solutions, determine the number of solutions to the following congruences.
 - (a) $8911x \equiv 83 \pmod{75911}$
 - (b) $21359x \equiv 26 \pmod{99619}$
8. How many solutions are there to the congruence $ax \equiv b \pmod{m}$? Explain your answer by turning this into a question about solving equations in $\mathbb{Z}$.
9. If a number is a perfect fourth power in $\mathbb{Z}_{29}$, how many fourth roots does it have? Use the ideas of logarithms and your answer to number 8.
10. If a number is a perfect fourth power in $\mathbb{Z}_{31}$, how many fourth roots does it have?