Math 290 Number Theory for Teachers
Homework 3
Due: February 5, 2014

1. There was a time in my life when I would carry around 4 or 5 pennies at all times so that when I paid cash for something, I could give the right number of pennies to make the cents I gave congruent mod 5 to the cents I owed. (That is, for example, if what I bought cost \$6.17, I would give some bills and 2 pennies since $17 \equiv 2 \mod 5$. Explain why I might have wanted to do this. What kind of change was I going to get back and why?

2. Which of the following can you find in $\mathbb{Z}_7$? (If you can find them, give a value or values.)

$$4 \cdot 5,\ -2,\ \frac{1}{2},\ \frac{2}{5},\ \sqrt{2},\ \sqrt{-1},\ \sqrt[3]{6}$$

3. Which of the following can you find in $\mathbb{Z}_{13}$? (Again, if you can find them, give a value or values.)

$$\frac{1}{2},\ \frac{2}{3},\ \frac{3}{5},\ \sqrt{-1},\ \sqrt{2}$$

4. Which of the following can you find in $\mathbb{Z}_6$? (Again, if you can find them, give a value or values.)

$$\frac{1}{2},\ \frac{2}{3},\ \frac{3}{5},\ \sqrt{-1},\ \sqrt{2}$$

5. Which fractions $\frac{a}{b}$ are in $\mathbb{Z}_m$? Explain.

6. List the elements of U_8, U_{11} and U_{13}.

7. How can you tell which elements of $\mathbb{Z}_m$ are in U_m? Explain.

For problems 8, 9, and 10 either prove the statement is true or give an example showing that it is false (a counterexample).

8. If a and b are elements of U_m, then $a + b$ is in U_m.

9. If a and b are elements of U_m, then $a \cdot b$ is in U_m.

10. If a is in U_m, then $-a$ is in U_m.