

MATH 290 NUMBER THEORY FOR TEACHERS

HOMEWORK 11

DUE: WEDNESDAY, APRIL 30, 2014

1. Show that finding pentagonal numbers that are also squares reduces to solving $x^2 - 6y^2 = 1$ with some restrictions on x and y . Be sure to say what these restrictions are.
2. Give an example of a solution to $x^2 - 6y^2 = 1$ that *doesn't* give a pentagonal number that is also a square.
3. Besides 1, find two other numbers that are both pentagonal and square.
4. If you complete the square for $ax^2 + bx + c$ (that is, manipulate this expression into the form $a(x + d)^2 + e$), what are the values of d and e in terms of a , b , and c ? Use your answer to derive the quadratic formula. Namely, show that if $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
5. Show that finding Pythagorean triples with consecutive legs can be turned into a problem about solving $x^2 - 2y^2 = -1$.
6. When finding Pythagorean triples whose legs are consecutive integers, can you tell whether the hypotenuse must be even or odd? Explain.
7. Explain how your answer to the previous problem restricts which solutions to $x^2 - 2y^2 = -1$ are useful in finding Pythagorean triples with consecutive legs.
8. Find three Pythagorean triples with consecutive legs.
9. Find three Pythagorean triples with one leg and the hypotenuse consecutive.