Math 290 Number Theory for Teachers
Homework 10
Due: Wednesday, April 16, 2014

1. Explain in your own words what it means when there is a large coefficient in the continued fraction expansion of a number and why. You may use an example to illustrate your point.

2. Use your explanation from problem 1 to explain why the golden ratio, $\frac{1+\sqrt{5}}{2}$ is "far" from rational in the sense that the convergents of its continued fraction expansion (the "best" rational approximations of the golden ratio) are not that close.

3. Play with the animation at the website below. Draw and briefly describe what happens to your flower/pinecone/seedhead in the animation when you put out a petal/seed at a rational number rotation (like .75 or .333333). Contrast this to what happens if you use an irrational number. (Note: the animation only lets you put in decimals, so you will never truly give it an irrational number. Just give it four or five of decimal places.) Why does having rotations of π give a bad flower? What numbers make the best flower and why?

`http://www.mathsisfun.com/numbers/nature-golden-ratio-fibonacci.html`

4. A particularly beautifully proportioned rectangle is said to be the one which, when you cut off a square, the remaining rectangle is similar to the original. Show that this rectangle must have the ratio of its length to its width equal to the golden ratio.

5. Approximate this beautiful rectangle by starting with a 1-by-1 square, attaching a 1-by-1 square to it, attaching a 2-by-2 square to that, then attaching a 3-by-3 square, etc. Attach your squares of Fibonacci side lengths in a spiral. Use this to sketch a convincing nautilus shell!

(Reference: `https://www.youtube.com/watch?v=ahXIMUkSXX0`)

6. List the first seven triangular numbers and find a formula for the nth triangular number.

7. List the first seven pentagonal numbers and find a formula for the nth pentagonal number.

8. Explain why if x and y satisfy $x^2 - 2y^2 = 1$, it must be the case that x is odd and y is even. Hint: think about the equation mod 4. What are the squares mod 4?

9. Show that finding triangular numbers that are also perfect squares is equivalent to solving $x^2 - 2y^2 = 1$. Hint: You will need to complete the square and use the previous problem.

10. Find three triangular numbers that are also perfect squares.