Math 290 Number Theory for Teachers
Homework 1
Due: January 22, 2014

1. Give an example of a number system or operation that is not commutative.

2. Show, with an example, that composition of functions is not necessarily associative.

3. Give an example explaining why the positive real numbers do not satisfy the well-ordering principle (WOP). In other words, give a subset of positive real numbers and explain why it does not have a smallest element.

4. In your own words, explain (the procedure and the justification of) the standard algorithm for multiplication of two two-digit numbers. Appeal to the axioms when appropriate.

5. Decide whether each statement is true or false. If true, explain why using the definition of divides.

i. $3 \mid 6$
ii. $6 \mid 2$
iii. $2 \mid 0$
iv. $0 \mid 1$
v. $1 \mid 5$

6. Justify each step of the following proof, using the axioms and definitions from class.

Claim: If $a \mid b$ and $a \mid c$, then $a \mid (b + c)$ for a, b, and c in $\mathbb{Z}$.

Proof:

There exist k and l in $\mathbb{Z}$ such that $ak = b$ and $al = c$.

Then $b + c = ak + al$.

So $b + c = a(k + l)$.

$k + l$ is in $\mathbb{Z}$.

So $a \mid (b + c)$.

7. Is the converse of the statement in problem 6 true or false? In other words, if $a \mid (b + c)$, is it true that $a \mid b$ and $a \mid c$? If it is true, prove it. If it is false, give an example demonstrating it is false (i.e. a counterexample).

8. Prove, using the axioms, that if $ab = ac$ and $a \neq 0$, then $b = c$. You may use that if $xy = 0$, then either $x = 0$ or $y = 0$ (or both). (Hint: You will need to use the distributive law.)

9. Give an example showing that the conclusion in problem 8 might be false if we don't include $a \neq 0$ in the assumptions.

10. Christina says that if a number is divisible by 3 and also by 7, then it is divisible by 21. Is she right? Irene then conjectures that for natural numbers a, b and c, if $a \mid c$ and $b \mid c$, then $ab \mid c$. Is she right? If so, justify. If not, give a counterexample and salvage Irene's claim by giving a similar true statement.