Math 290-Number Theory for Teachers
Problem of the Day #15
Due Friday, March 14, 2014

1. Recently, someone told me (Li-Mei) about a trick he had been taught in elementary school for checking your work when doing multi-digit multiplication problems. For each factor, you add the digits repeatedly until you get down to a one-digit number from each factor. You multiply these digits, do the repeated sum process and compare to the sum of the digits of the product you got (again, possibly adding the digits multiple times), and if these agree, probably you're right, but if not, you have an error.

For example, say you computed $1934 \times 657 = 1260638$. You would check your work by computing $1 + 9 + 3 + 4 = 17$ and then $1 + 7 = 8$ for the first factor. For the second factor, you'd get $6 + 5 + 7 = 18$ and $1 + 8 = 9$. Multiplying these, we get $8 \times 9 = 72$, which, when we take the sum of the digits, gives us $7 + 2 = 9$. We then check that $1 + 2 + 6 + 0 + 6 + 3 + 8 = 26$ and $2 + 6 = 8 \neq 9$. Therefore, we know we have an error!

Do you believe that this method of checking your work is valid? Why? (Try some examples to convince yourself, and think about what summing the digits of a number tells us about the original number mod 9.) Are there any errors that this method won't catch?