Math 290-Number Theory for Teachers
Problem of the Day #11
Due Wednesday, February 19, 2014

We call an element a of U_m a *generator* if every element of U_m can be expressed as a power of a. For example, 3 is a generator for U_7 since $3^1 = 3,\ 3^2 = 2,\ 3^3 = 6,\ 3^4 = 4,\ 3^5 = 5,\ 3^6 = 1$ (all mod 7), so we get all the elements of U_7, 1, 2, 3, 4, 5, and 6, as powers of 3.

1. Look at U_3, U_4, U_5, U_6, U_7, U_8, U_9, U_{11}, U_{12}, U_{13}. Which of these have a generator? Any conjectures?

2. For elements that are not generators, what do you notice about their *orders*? (The order of an element a in U_m is the smallest natural number n such that $a^n \equiv 1 \mod m$.)