MATH 191 – PRACTICE MIDTERM 1

Name: SOLUTIONS

FOR FULL CREDIT, SHOW ALL WORK
NO CALCULATORS

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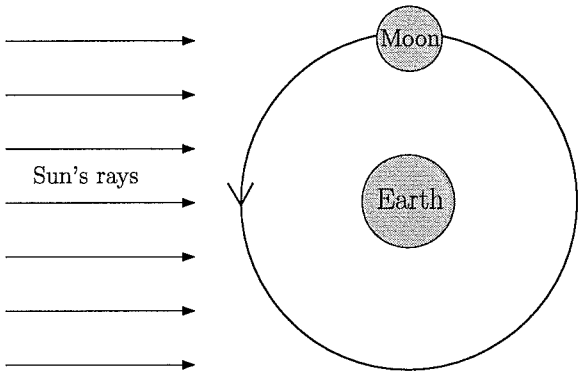
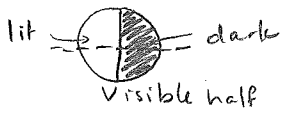
1. Look at the figure below and use it to answer the following questions. Be sure to clearly explain all your answers, using diagrams when appropriate.



(a) What does the moon look like to people on Earth who can see it?

The moon looks like ◑ to people on Earth who can see it. In the diagram, the left half of the moon is illuminated, but people on Earth only see the bottom half of the moon.



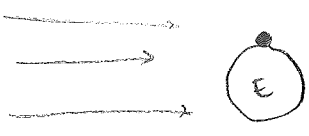
Therefore, they see a half moon with the left half lit.

(b) Is the moon waxing or waning?

The moon is waning. It was a full moon when the moon was on the far side of the earth from the Sun, and the moon is moving toward a position where it is a new moon (between the Earth and sun). Therefore, since the moon is going from full to new, it is getting smaller, or waning.

(c) Assuming that in the figure, we are looking down on the North Pole, if you see the moon high in the sky, what time is it?

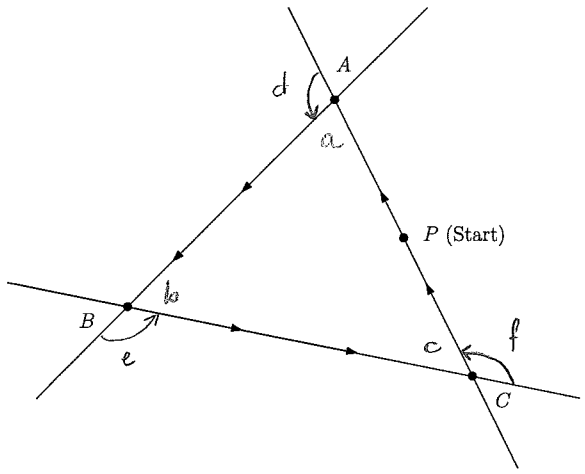
To see the moon high in the sky, you must be standing on the side of the Earth directly facing the moon.



Then North is up out of the page for you and the sun is coming from the east (your right if you're facing north).

Therefore, it is morning, around sunrise.

2. Use the figure below to answer the following questions.



(a) If you start standing at point P facing point A and walk along the path indicated to return to P, which angles do you turn? Indicate the angles on the diagram.

(b) When you return to point P, how many degrees have you turned? (You can answer this question without measuring the angles.)

When you return to point P, you have turned 360°. You end facing the same way you started and you made one full rotation.

(c) Use your answers to explain why the sum of the angles in a triangle is 180°.

Let the angles be labeled as indicated in the diagram.

From part b, $d+e+f = 360°$

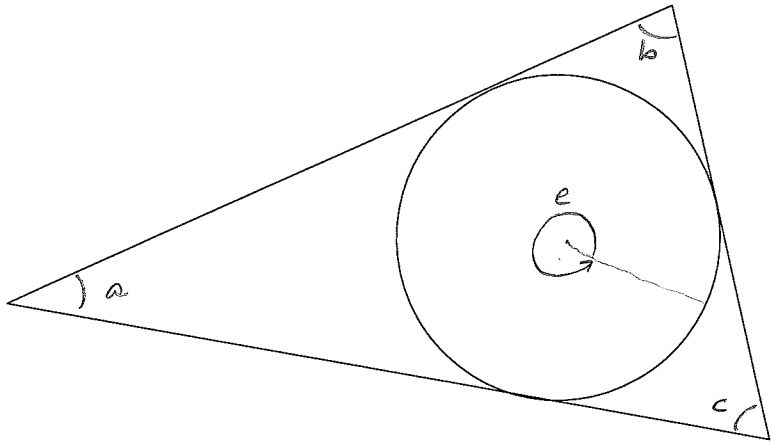
But also $b+e = c+f = a+d = 180°$ since each of these pairs of angles is adjacent and add to a straight line.

So $a = 180-d$, $b = 180-e$, $c = 180-f$ and

$$a+b+c = 180-d+180-e+180-f = 540-(d+e+f)$$
$$= 540-360 = 180°$$

Thus the sum of the angles in this triangle ($a+b+c$) is 180°. This process would have worked for any triangle and doesn't depend, for example, on the side lengths.

3. Johnny looks at the diagram below and says that because the circle is inside the triangle, the circle must have fewer degrees than the triangle. Lindsay counters that a triangle has 180° and a circle has 360°, so Johnny is wrong. Explain why Johnny is wrong. What could Johnny compare instead of angle measures and say that the circle is smaller than the triangle?

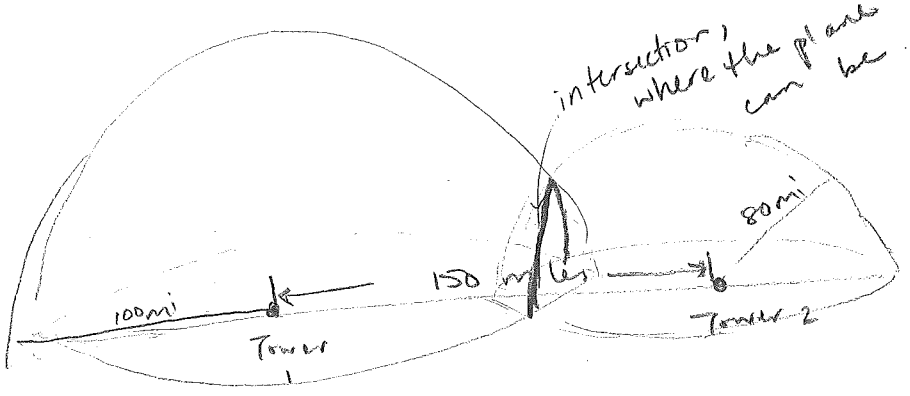


Johnny is comparing the size or area of the shapes, not the angle measures.

When he says "the angles in the triangle," he's referring to the angles a, b and c labeled in the diagram. But when we say there are 360° in a circle, we mean that it takes 360° of rotation to make a full rotation, as indicated at e. The sum of angles a, b, and c has no relation to the angle at e, so Johnny is wrong to compare them.

4. An air traffic control tower detects a plane 100 miles away. A second air traffic control tower 150 miles from the first detects the same plane 80 miles away. Describe all possible locations of the plane, and explain your answer.

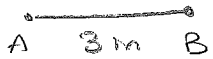
All possible locations of the plane are the intersection points of two spheres, one with center at the first tower and radius 100 miles and the second with center at the second tower and radius 80 miles. If the plane is 100 miles from the first tower, it is on the sphere of radius 100 miles centered at the first tower. since that sphere is the set of all points 100 miles from the tower. If the plane is 80 miles from the second tower, it is on the sphere of radius 80 miles centered at the second tower. Thus, for the plane to be 100 miles from the first tower AND 80 miles from the second, it must be on both spheres, i.e. on their intersection.



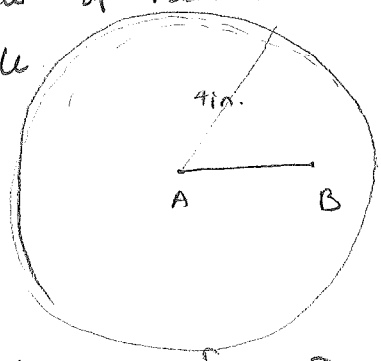
(Note: since the plane isn't underground, we could actually say the plane is on the intersection of two half spheres).

5. Explain how you would construct a triangle with side lengths 3 inches, 4 inches and 5 inches using a ruler and compass. Be sure to explain why your construction works. If you know that the triangle is a right triangle, how might you construct the triangle using a piece of paper, a ruler, and scissors?

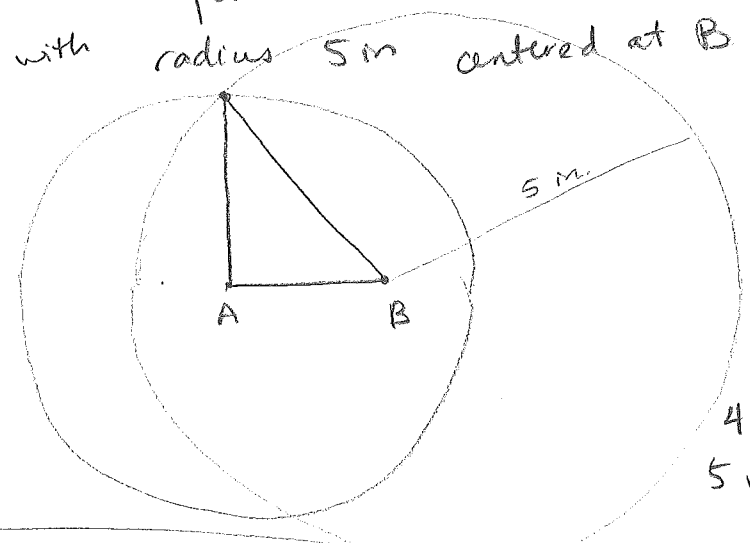
Start by drawing a line segment of length 3 inches
This gives us one side of the triangle and 2 vertices



Then we want to find the 3RD vertex. It should be
4 in from A and 5 in from B.
The set of points 4 in from A are the points making up the circle of radius 4 in. centered at A.
Draw that circle

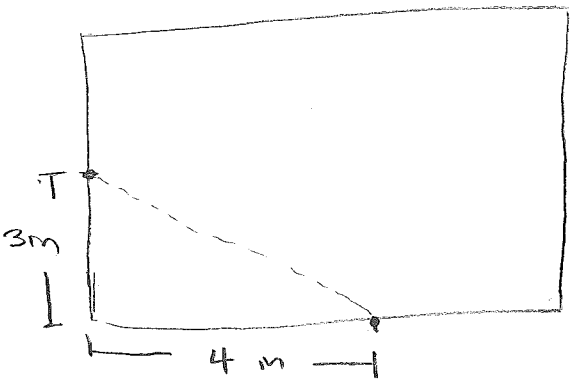


The set of points 5 in from B are the points of the circle with radius 5 in centered at B Draw that circle



Choose one of the intersection points. That is the 3RD vertex since it must be 4 in from A and 5 in from B.

To construct the triangle with paper, a ruler and scissors, we could measure 3 in along one side of the paper, 4 in along the other as shown in the diagram



Then we could connect the points on the edge of the paper and cut along that line.

If we know the triangle is a right triangle, the last side should be 5 m.

We can verify by measuring.

6. a. Using the definition that a that a trapezoid has *at least* one pair of parallel sides, explain in words the relationship between parallelograms and trapezoids.

   b. What is the relationship between squares and rhombuses? Explain.

a. Every parallelogram is a trapezoid with this definition. A parallelogram has two pairs of parallel sides, which is at least one pair. Therefore, any parallelogram is a trapezoid

b. Every square is a rhombus. Rhombuses are quadrilaterals with all four sides the same length. Squares are quadrilaterals with all four sides the same length and all four angles equal to 90°. The requirements for squares are more restricted. Every square automatically satisfies the condition to be a rhombus.