

MATH 191 FUNDAMENTALS OF MATHEMATICS II  
8.2: EVEN AND ODD, 8.3: DIVISIBILITY TESTS, AND 8.4 PRIME NUMBERS  
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**Even and Odd**

We say a \_\_\_\_\_ is \_\_\_\_\_ if

- it is \_\_\_\_\_ OR
- it can be \_\_\_\_\_ as \_\_\_\_\_  
OR
- you can divide that number of objects into \_\_\_\_\_  
with \_\_\_\_\_ OR
- you can divide that number of objects into \_\_\_\_\_  
with \_\_\_\_\_.

These definitions are all \_\_\_\_\_, meaning that if one holds, they all do.

On the other hand, we say a \_\_\_\_\_ is \_\_\_\_\_ if

- it is \_\_\_\_\_ OR
- it can be \_\_\_\_\_ as \_\_\_\_\_  
OR
- you can divide that number of objects into \_\_\_\_\_  
with \_\_\_\_\_ OR
- you can divide that number of objects into \_\_\_\_\_  
with \_\_\_\_\_.

How do we tell if a number is even or odd?

Why does this work?

Imagine representing a counting number with \_\_\_\_\_. For example, 6531 is represented by

Then we can \_\_\_\_\_ each bundle into groups of two. We do this for the bundles of 10, 100, 1000 and so on.

Therefore, we see that the powers of 10 bundles can always be divided evenly into groups of twos, and the original full number of sticks can be divided evenly into groups of twos exactly when

## Divisibility Tests

That was an example of a \_\_\_\_\_, that is, a way of telling if a number is divisible by another.

**Divisibility test for 2:** A number is divisible by 2 if

**Divisibility test for 10:** A number is divisible by 10 if

**Divisibility test for 5:** A number is divisible by 5 if

The explanations for the divisibility tests for 10 and 5 are similar to the way we tested for whether a number is even.

**Divisibility test for 3:** A number is divisible by 3 if

Example: This test tells us that 248 is not divisible by 3 because  $2 + 4 + 8 = 14$  is not divisible by 3.

Why does this work? Again think of our number as being represented by base-ten bundles of sticks. So 248 is represented by

Now if we try to divide these bundles into \_\_\_\_\_, the bundles of 10 will each

The bundles of 100 will each

If we combine the leftover sticks from each bundle with the individual sticks from the ones place, we have \_\_\_\_\_ in total. If these sticks can \_\_\_\_\_, the original number is \_\_\_\_\_, and if not, \_\_\_\_\_.

This works for any number because the bundles of powers of ten will always have one stick left over when divided into groups of three.

A more algebraic explanation is, if the number we're testing is  $ABCD$ ,

This tells us that  $ABCD$  is divisible by 3 if \_\_\_\_\_ and  
it is not divisible by 3 if \_\_\_\_\_.

**Divisibility test for 9:** A number is divisible by 9 if

**Divisibility test for 4:** A number is divisible by 4 if

## Prime Numbers

A \_\_\_\_\_ is a \_\_\_\_\_ other than \_\_\_\_\_  
that are \_\_\_\_\_. The opposite of prime is  
\_\_\_\_\_, and a \_\_\_\_\_ is a counting  
number other than \_\_\_\_\_ that is \_\_\_\_\_.

Primes are the building blocks of the counting numbers because

In fact, every counting number greater than 1

This is called the \_\_\_\_\_.

We can find prime numbers using the Sieve of Eratosthanes. The method is:

(1) \_\_\_\_\_ the number \_\_\_\_\_, and \_\_\_\_\_  
every second number after that.

(2) \_\_\_\_\_ the \_\_\_\_\_  
which is \_\_\_\_\_. Cross out every \_\_\_\_\_ after 3.

(3) \_\_\_\_\_ the \_\_\_\_\_  
and cross out every \_\_\_\_\_ after this.

(4) Repeat this process, circling the \_\_\_\_\_  $N$  that \_\_\_\_\_  
and cross out every \_\_\_\_\_ number after  $N$ , until every number has either been  
circled or crossed out.

The circled numbers are the primes.

|    |    |    |    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|----|----|----|
|    | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |