Math 191 Fundamentals of Mathematics II
8.7 Number Systems and 8.6 Rational and Irrational Numbers
April 14, 2014

Number Systems Overview

The first numbers we encounter are the ____, used to ___________________________ objects:

Then we might need to talk about having ___________________________ of an object, and introduce the concept of ___________________________ to get the ____:

Although we can do ___________________________ with whole numbers and always get whole number answers, we cannot do the same with ___________________________.
For example, ____ is not a whole number. To accommodate problems that use these calculations, we expand to the ____:

We can add, subtract, and multiply integers and our answer will always be an integer, but this isn't true for ___________________________. For example, ____ is not an integer. To accommodate problems that use these calculations, we expand to the ____, that is, all numbers that can be expressed as a ___________________________ or the ____.
Examples:

There are still some problems we can't solve using the rational numbers. For example, the solutions to ____ are not rational. To solve these problems, we expand finally to the ____, the numbers which can be expressed with ____.

We can use a Venn diagram to show the relationship between these number systems:

Expressing Rational Numbers as Decimals

Let's look more in-depth at rational numbers, that is numbers that can be expressed as a ____________________________.

Note: Even when we express a fraction as a ____________________________, it is still rational. Example:

To determine how to express a fraction as a decimal, we use ____. For example, express $\frac{1}{8}$ and $\frac{2}{3}$ as decimals.

We say a decimal is __________________________ if it has a __________________________ __________________________ that ____. The repeating part doesn't have to start right after the decimal place. We write a __________________________ over the _______________________________________ to distinguish it.

We say a decimal is __________________________ if it has only __________________________ __________________________ non-zero digits in its expansion. We can think of these decimals as repeating, but with the repeating digit being _____________. For example,

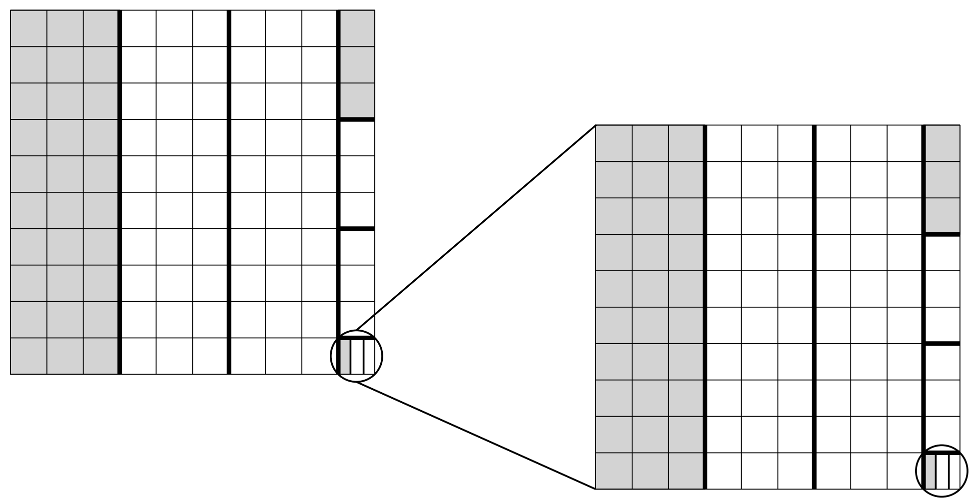
Every fraction has a decimal expansion that is either _______________________________________ or _______________________________________. This is because, when we do the long division we either

- get a remainder of __________________________ at some point. In this case, the decimal __________________________.
- get a remainder that _______________________________________ at some earlier point. This is because, if the denominator is B, the only remainders we can have when dividing by B are ____. If we have infinitely many non-zero remainders, we must __________________________ a remainder, and the decimal expansion _______________________________________ at this point.

A number whose decimal doesn't repeat or terminate is __________________________. For example, 0.12112111211112... is irrational.

Representing Decimal Representations with Subdivided Squares

Suppose the large square represents 1, and we are trying to see $\frac{1}{3}$.



Writing a Terminating or Repeating Decimal as a Fraction

We can write a terminating decimal as a fraction by using a ______________________ that is a ____. For example, 0.12345 can be written as

To write a repeating decimal N as a fraction, we

- ______________________ by a __________________________________ if necessary to get the repeating part to start right after the decimal place.
- ______________________ by another __________________________________ so that we have the same repeating part after the decimal.
- ______________________ these multiples of N from each other to get a ____________ ____________. Note that we have chosen the multiples of ten so that the ____________ ____.
- Now we know that a particular multiple of N is equal to a whole number, and dividing by this coefficient gives us the value of N.

For example, if $N = 0.123123123... = 0.\overline{123}$,

If $N = 0.8234343434... = 0.82\overline{34}$,