Math 191 Fundamentals of Mathematics II
8.4: Prime Numbers, 8.5: Greatest Common Factor and Least Common Multiple
April 11, 2014

Prime Numbers: The Trial Division Method

We want a way of telling if a number (such as 181) is prime.

Method 1: Use ____ to find all the primes up to 181.

Method 2: __________________________. This method says: __________ the number by __________. If it __________________________, try dividing the number by __________. Continue trying to divide by __________________________. If you ____________________________________ that __________________ the number evenly, the number ____________________________________. Otherwise, __________________________.

Notice that at a certain point the ______________ become ____________________ ____________________. In our example, this occurs when the divisor is __________. We can stop at this point because

The Fundamental Theorem of Arithmetic says

We can determine ____

using a ____________________________________.

- Step 1: ________________________ the number in ____________________________________ you can think of.
- Step 2: ________________________ each of the factors individually.
- Step 3: Continue factoring in this way until all the factors are __________________ and cannot be factored further.

Example: Factor 500 into a product of primes.

If you made a different factor tree for 500, would you end up with the same prime factors? If so, why?

If we don't immediately see a factorization for our number, we can use ________________________ to systematically find a prime factor.

Greatest Common Factor and Least Common Multiple

When you have ____________________________________ counting numbers, their ____ (sometimes called the ____) is the

For example, what is the GCF of 16 and 24?

The ____ of two or more counting numbers is the

For example, what is the LCM of 16 and 24?

How to find GCFs and LCMs

Method 1: ____

In this method, we ____ and ____.

Example: Find the GCF and LCM of 6600 and 5200.

Why does this method give the GCF? We took out common factors until the only common factor was 1. By keeping track of the successive quotients, we make sure we don't double count any common factors. But since we took out all the common factors, multiplying them all together should give us the largest common factor possible.

Why does this method give the LCM? The product of the common factors we took out with one of the quotients at the bottom gives us one of the original numbers. So the product of the common factors with both quotients gives us a multiple of both of the original numbers. Since we don't repeat any of the common factors in this product, this is the smallest common multiple.

Method 2: Using the ____

- ______________________ each number into primes.
- The GCF is the product of each of the primes raised to the ______________________ power appearing in the factorizations.
- The LCM is the product of each of the primes raised to the ______________________ power appearing in the factorizations.

Example: Find the GCF and LCM of $2^6 \cdot 3^2 \cdot 7^3$ and $2^2 \cdot 3^4 \cdot 5 \cdot 11$.

Problems Involving GCFs and LCMs

1. Mary will make an 8-inch-by-12-inch quilt for a doll house out of identical square patches. Each square patch must have sides that are a whole number of inches long and no partial squares are allowed. What size can the squares be? What is the largest the squares can be?

2. Two gears are meshed, with marked teeth aligned. One gear is smaller and has 15 teeth. The other gear is larger and has 36 teeth. How many revolutions will the smaller gear make for the marked teeth to be aligned again?

3. When do we use GCFs in fraction problems?

For example,

4. When do we use LCMs in fraction problems?

For example,