Math 191 Fundamentals of Mathematics II
12.9: The Pythagorean Theorem
February 28, 2014

What the Pythagorean Theorem Says

The Pythagorean Theorem is named for the ancient Greek mathematician _____ (who lived around 500 BC), but may have been known long before his time, possibly as far back as the ancient Babylonians (2000 BC).

The Pythagorean theorem is a statement about all _____. It says that the _____ is equal to the _____. In other words,

Note that this theorem only holds for _____ and the side lengths must be _____.

Proofs of the Pythagorean Theorem

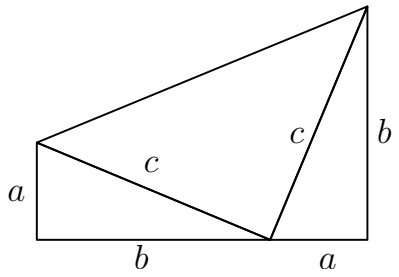
There are many proofs of the Pythagorean Theorem (hundreds!) but this first one is attributed to Pythagoras.

1. In your groups, try to arrange the right triangles and squares to make two shapes of equal area. Each shape should use four triangles. Use your arrangements to discuss why $a^2 + b^2 = c^2$ if a, b, and c are the side lengths of the triangle (with c being the length of the hypotenuse). Some things to think about:

- What does the quantity a^2 represent? What about b^2 and c^2?
- Why are the shapes you made of the same area?
- How does that tell you that $a^2 + b^2 = c^2$?

This next proof is due to President Garfield. It uses some algebra.

2. Look at the figure below. What shape is it? (You may assume that the triangles on the left and right are both right triangles.)



By computing the area of this shape in two different ways, prove that $a^2 + b^2 = c^2$. Consider:

- The triangle in the middle (with side length c) is a right triangle. Why?
- By looking at the areas of these three triangles individually, what is the area of the whole shape?
- By looking at the type of shape this is and what you know about finding its area, what is the area of the shape?