Math 6 – Practice Midterm 1

Name: SOLUTIONS

For Full Credit

Show All Work

No Calculators

1. Find the area under the curve defined by $y = \sqrt[3]{x+1}$ from $x = 0$ to $x = 7$.

Area $= \int_0^7 \sqrt[3]{x+1}\, dx$

Set $u = x+1$. Then $du = dx$.

If $x = 0$, $u = 1$, and if $x = 7$, $u = 8$.

Then

$$\text{Area} = \int_1^8 \sqrt[3]{u}\, du = \frac{3}{4} u^{4/3} \Big|_1^8 = \frac{3}{4}\left(8^{4/3} - 1^{4/3}\right)$$

$$= \frac{3}{4}(16-1) = \boxed{\frac{45}{4}}$$

2. Compute the indefinite integral:

$$\int \frac{1}{(2\sqrt{x})^3} dx$$

$$\int \frac{1}{(2\sqrt{x})^3}\, dx = \int \frac{1}{8x^{3/2}}\, dx$$

$$= \frac{1}{8} \int x^{-3/2}\, dx$$

$$= \frac{1}{8} \frac{x^{-1/2}}{-1/2} + C$$

$$= \boxed{\frac{-1}{4\sqrt{x}} + C}$$

3. Compute the following definite integral:

$$\int_1^2 \frac{e^{1/x}}{x^2}\,dx$$

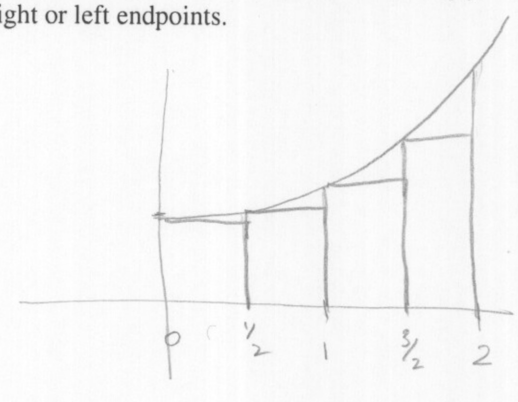
Set $u = \frac{1}{x}$. Then $du = -\frac{1}{x^2}\,dx$ and $-du = \frac{1}{x^2}\,dx$

Change limits: If $x=1$, $u=1$. If $x=2$, $u=\frac{1}{2}$.

So: $\int_1^{1/2} -e^u\,du = -e^u\Big|_1^{1/2} = -e^{1/2} - (-e)$

$= \boxed{e - \sqrt{e}}$

4. Estimate the area under the curve defined by $y = e^{(x^2)}$ from $x = 0$ to $x = 2$ using **four** rectangles. You may use right or left endpoints.



Using left end points:

Area $\approx \frac{1}{2}\cdot e^0 + \frac{1}{2}e^{1/4} + \frac{1}{2}e^1 + \frac{1}{2}e^{9/4}$

$= \boxed{\frac{1}{2}\left(1 + e^{1/4} + e + e^{9/4}\right)}$

Using right endpoints:

Area $\approx \frac{1}{2}e^{1/4} + \frac{1}{2}e^1 + \frac{1}{2}e^{9/4} + \frac{1}{2}e^4$

$= \boxed{\frac{1}{2}\left(e^{1/4} + e + e^{9/4} + e^4\right)}$

5. A particle has velocity given by $v(t) = t^3 + 2t + 3$. At time $t = 0$, it is at position $y = 1$. Find a formula giving the position $y(t)$ of the particle.

$$y(t) = \int v(t)\,dt = \int t^3 + 2t + 3 \;dt = \frac{t^4}{4} + \frac{2t^2}{2} + 3t + C.$$

$$y(0) = 1 = \frac{0^4}{4} + 0^2 + 3\cdot 0 + C \Rightarrow C = 1.$$

So $\boxed{y(t) = \frac{t^4}{4} + t^2 + 3t + 1}$

6. Define $F(x) = \int_0^x \frac{t}{t^4+1}\,dt$.

(a) What is $F'(x)$?

$\boxed{F'(x) = \frac{x}{x^4+1}}$ (Second fundamental theorem of calculus).

(b) At what values of x could $F(x)$ have maxima, minima, and points of inflection?

$F(x)$ has maxima, minima and points of inflection when $F'(x) = \frac{x}{x^4+1} = 0$.

$F'(x) = 0$ if $\boxed{x = 0}$

(c) Is $F(1)$ positive, negative or zero?

$F(1)$ is $\boxed{\text{positive}}$ because $\frac{t}{t^4+1}$ is positive from 0 to 1. Since $F(x)$ measures the area between the graph $\frac{t}{t^4+1} = y$ and the horizontal axis, and this area is above the axis, $F(1)$ is positive.