MATH 6 – MIDTERM 2

Name: SOLUTIONS

FOR FULL CREDIT

SHOW ALL WORK

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NO CALCULATORS

1. Compute the following definite integral.

$$\int_0^3 \frac{2x^3+x^2+1}{2x+1}\,dx$$

(10 points)

Start with long division:

$$\begin{array}{r} x^2 \qquad\qquad\quad \\ 2x+1 \enclose{longdiv}{2x^3+x^2+0x+1} \\ -\underline{(2x^3+x^2)} \qquad\quad \\ 1 \end{array}$$

So we have $\int_0^3 x^2 + \frac{1}{2x+1}\,dx = \int_0^3 x^2\cancel{dx+}\int_0^3 \frac{1}{2x+1}\,dx$

$u = 2x+1$
$du = 2dx$, so
$\frac{1}{2}du = dx$

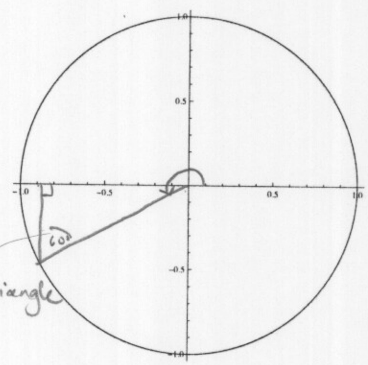
$$= \frac{x^3}{3}\Big|_0^3 + \frac{1}{2}\int_{x=0}^{x=3} \frac{1}{u}\,du$$

$$= \left(\frac{27}{3} - 0\right) + \frac{1}{2}\ln|u|\Big|_{x=0}^{x=3} = 9 + \frac{1}{2}\ln|2x+1|\Big|_0^3$$

$$= 9 + \frac{1}{2}\ln 7 - \frac{1}{2}\ln 1$$

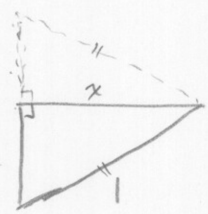
$$= \boxed{9 + \frac{1}{2}\ln 7}$$

2. Indicate on the unit circle below where the angle $\frac{7\pi}{6}$ is. Then, evaluate $\sin\frac{7\pi}{6}$, $\cos\frac{7\pi}{6}$ and $\tan\frac{7\pi}{6}$.



30-60-90 triangle

(10 points)



$$x^2 + \left(\frac{1}{2}\right)^2 = 1$$

$$x^2 + \frac{1}{4} = 1$$

$$x = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}.$$

$$\sin\frac{7\pi}{6} = -\frac{1}{2}$$

$$\cos\frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$$

$$\tan\frac{7\pi}{6} = \frac{-1/2}{-\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}.$$

3. Solve the following *separable* differential equation, with initial condition $y(0) = 0$.

$$\frac{dy}{dx} = e^{x+y}$$

(10 points)

$$\frac{dy}{dx} = e^x \cdot e^y$$

Separate $\quad e^{-y}\, dy = e^x\, dx$

Integrate: $\quad \int e^{-y}\, dy = \int e^x\, dx$

$$-e^{-y} = e^x + C_1$$

So $\quad e^{-y} = -e^x + C_2$

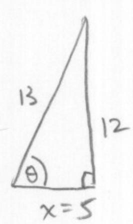
$$-y = \ln(-e^x + C_2)$$

$$y = -\ln(-e^x + C_2)$$

$0 = -\ln(-e^0 + C_2) = -\ln(-1 + C_2)$, so $C_2 = 2$

$$\boxed{y = -\ln(-e^x + 2)}$$

4. Given that $\sin\theta = \frac{12}{13}$ and θ is an acute angle, find $\cos\theta$, $\tan\theta$, $\sec\theta$, $\cot\theta$ and $\csc\theta$. (10 points)



$$13^2 = 12^2 + x^2$$
$$169 = 144 + x^2$$
$$25 = x^2$$
So $x = 5$

$\cos\theta = \frac{adj}{hyp} = \frac{5}{13}$ $\qquad$ $\sec\theta = \frac{1}{\cos\theta} = \frac{13}{5}$

$\tan\theta = \frac{opp}{adj} = \frac{12}{5}$ $\qquad$ $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{5}{12}$

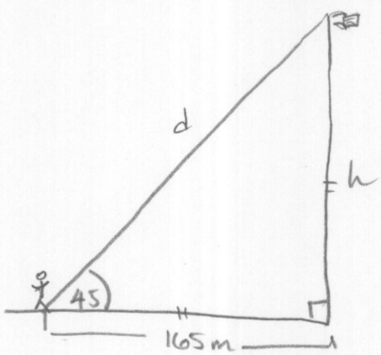
$\csc\theta = \frac{1}{\sin\theta} = \frac{13}{12}$

5. What is the *angular velocity* in radians per minute of a ferris wheel with diameter 200 feet which makes 3 rotations per minute? (5 points)

$$\frac{3 \text{ rotations}}{\text{minute}} \times \frac{2\pi \text{ radians}}{\text{rotation}} = \boxed{6\pi \ \frac{\text{radians}}{\text{minute}}}$$

6. The tallest freestanding flagpole in the world, the Dushanbe Flagpole in Tajikistan, has a 165 meter shadow when the sun has an angle of elevation of 45°.

(a) What is the height of the flagpole? (5 points)



The flag pole is $\boxed{165 \text{ m}}$ tall.
(45-45-90 triangle).

Alternately, $\frac{h}{165} = \tan 45° = 1$, so $h = 165$.

(b) If you stand at the tip of the shadow, how far are you from the top of the flagpole? (5 points)

$$\frac{165}{d} = \cos 45° = \frac{\sqrt{2}}{2}, \text{ so } \quad d = 165 \cdot \frac{2}{\sqrt{2}} = \boxed{165\sqrt{2} \text{ m}}$$

7. The rate of change of the number of bacteria in a culture is proportional to the current number of bacteria present.

(a) Write a differential equation to model this. Define all your "variables" for full credit. (5 points)

Let $P(t)$ be the number of bacteria present at time t.

$\frac{dP}{dt} = kP$, where k is a constant.

(b) Solve the differential equation you wrote in part (a). (That is, find the general solution to your differential equation.) (10 points)

$\frac{dP}{dt} = kP$ is separable.

$$\frac{dP}{P} = k\,dt$$

Integrate both sides:

$$\int \frac{dP}{P} = \int k\,dt$$

$$\ln|P| = kt + C_1$$

$$|P| = e^{kt+C_1} = e^{kt} \cdot e^{C_1}$$

So $\boxed{P = Ce^{kt}}$.