MATH 6 – QUIZ 4
2 MARCH 2012

Name: SOLUTIONS

NO CALCULATORS

1. Compute the following definite integral.

$$\int \frac{2x^2+x+4}{x^2+2}\,dx$$

$$\begin{array}{r} 2 \\ x^2+2 \overline{)\ 2x^2+x+4} \\ \underline{2x^2\ \ \ +4} \\ x \end{array}$$

$$\int 2 + \frac{x}{x^2+2}\,dx = 2x + \int \frac{x}{x^2+2}\,dx$$

Let $u = x^2+2$.
$du = 2x\,dx$, so $\frac{1}{2}du = x\,dx$.

$$\int \frac{x}{x^2+2}\,dx = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln(x^2+2) + C.$$

So the original integral is equal to $\boxed{2x + \frac{1}{2}\ln(x^2+2) + C}$

2. Solve the following differential equation.

$$\frac{dy}{dx} = y\sqrt{x} + 2y$$

$$\frac{dy}{dx} = y(\sqrt{x}+2)$$

$$\frac{dy}{y} = (\sqrt{x}+2)\,dx$$

Integrate both sides:

$$\int \frac{dy}{y} = \int (\sqrt{x}+2)\,dx$$

$$\ln|y| = \frac{2}{3}x^{3/2} + 2x + C_1.$$

So $\boxed{y = Ce^{\frac{2}{3}x^{3/2}+2x}}$