Homework 9 Solutions

Section 10.2

Odds

17. $\dfrac{\tan^2\theta}{\sec\theta} = \sin\theta\tan\theta$.

Start with the left side:

$$\frac{\tan^2\theta}{\sec\theta} = \frac{\left(\frac{\sin^2\theta}{\cos^2\theta}\right)}{\frac{1}{\cos\theta}} = \frac{\sin^2\theta}{\cos^2\theta}\cdot\cos\theta = \frac{\sin^2\theta}{\cos\theta} = \sin\theta\tan\theta.$$

So $\dfrac{\tan^2\theta}{\sec\theta} = \sin\theta\tan\theta$. ✓

55. $\tan^5 x = \tan^3 x\sec^2 x - \tan^3 x$

Start with right side:

$$\begin{aligned}\tan^3 x\sec^2 x - \tan^3 x &= \tan^3 x(\sec^2 x - 1)\\ &= (\tan^3 x)(\tan^2 x)\\ &= \tan^5 x.\end{aligned}$$

So $\tan^5 x = \tan^3\sec^2 x - \tan^3 x$. ✓

Evens

14. $\cos^2\beta - \sin^2\beta = 2\cos^2\beta - 1$

Start with left side:

$$\begin{aligned}\cos^2\beta - \sin^2\beta &= \cos^2\beta - (1-\cos^2\beta)\\ &= 2\cos^2\beta - 1\end{aligned}$$

So $\cos^2\beta - \sin^2\beta = 2\cos^2\beta - 1$ ✓

24. $\dfrac{\sec\theta - 1}{1-\cos\theta} = \sec\theta$.

Start with the left side

$$\frac{\sec\theta-1}{1-\cos\theta} = \frac{\frac{1}{\cos\theta} - 1}{1-\cos\theta} = \frac{\frac{1-\cos\theta}{\cos\theta}}{1-\cos\theta} = \frac{1}{\cos\theta} = \sec\theta.$$

So $\dfrac{\sec\theta - 1}{1-\cos\theta} = \sec\theta$. ✓

56. $\sec^4 x\tan^2 x = (\tan^2 + \tan^4 x)\sec^2 x$

Start with left side:

$$\begin{aligned}\sec^4 x\tan^2 x &= \sec^2 x(\sec^2 x)\tan^2 x\\ &= \sec^2 x(1+\tan^2 x)\tan^2 x\\ &= (\tan^2 x + \tan^4 x)\sec^2 x.\end{aligned}$$

So $\sec^4 x\tan^2 x = (\tan^2 x + \tan^4 x)\sec^2 x$.

58. $\sin^4 x + \cos^4 x = 1 - 2\cos^2 x + 2\cos^4 x.$

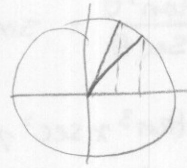
Start with the left side:

$$\sin^4 x + \cos^4 x = (\sin^2 x)^2 + \cos^4 x$$
$$= (1-\cos^2 x)^2 + \cos^4 x$$
$$= 1 - 2\cos^2 x + \cos^4 x + \cos^4 x$$
$$= 1 - 2\cos^2 x + 2\cos^4 x.$$

So $\sin^4 x + \cos^4 x = 1 - 2\cos^2 x + 2\cos^4 x$ ✓.

Section 10.4

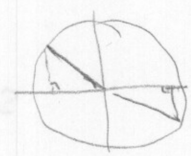
Odds



7. a) $\cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right) = \cos\frac{\pi}{4}\cos\frac{\pi}{3} - \sin\frac{\pi}{4}\sin\frac{\pi}{3}$

$$= \frac{\sqrt{2}}{2}\cdot\frac{1}{2} - \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}$$
$$= \frac{\sqrt{2}-\sqrt{6}}{4}$$

b) $\cos\frac{\pi}{4} + \cos\frac{\pi}{3} = \frac{\sqrt{2}}{2} + \frac{1}{2} = \frac{\sqrt{2}+1}{2}.$

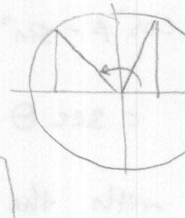


11. a) $\sin(135° - 30°) = \sin 135° \cos(-30°) + \sin(-30°)\cos(135°)$

$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{-1}{2}\cdot -\frac{\sqrt{2}}{2}$$
$$= \frac{\sqrt{6}+\sqrt{2}}{4}$$

b) $\sin 135° - \cos 30° = \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} = \frac{\sqrt{2}-\sqrt{3}}{2}$

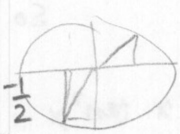
21. $\theta = \frac{13\pi}{12} = \frac{9\pi}{12} + \frac{4\pi}{12} = \frac{3\pi}{4} + \frac{\pi}{3}.$



$$\sin\frac{13\pi}{12} = \sin\frac{3\pi}{4}\cos\frac{\pi}{3} + \sin\frac{\pi}{3}\cos\frac{3\pi}{4}$$
$$= \frac{\sqrt{2}}{2}\cdot\frac{1}{2} + \frac{\sqrt{3}}{2}\cdot -\frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$$

$$\cos\frac{13\pi}{12} = \cos\frac{3\pi}{4}\cos\frac{\pi}{3} - \sin\frac{3\pi}{4}\sin\frac{\pi}{3}$$
$$= -\frac{\sqrt{2}}{2}\cdot\frac{1}{2} - \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} = \boxed{\frac{-\sqrt{6}-\sqrt{2}}{4}}$$

25. $\theta = 285° = 240° + 45°$

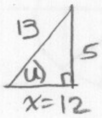


$$\sin 285° = \sin 240° \cos 45° + \sin 45° \cos 240° = \frac{-\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{-1}{2}$$
$$= \boxed{\frac{-\sqrt{6}-\sqrt{2}}{4}}$$

$$\cos 285° = \cos 240° \cos 45° - \sin 240° \sin 45° = -\frac{1}{2}\cdot\frac{\sqrt{2}}{2} - -\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$$

43. $\sin(u+v) = \sin u \cos v + \cos u \sin v$

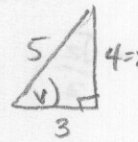
Given $\sin u = 5/13$ and $\cos v = -3/5$ and u, v in quadrant II.



$x^2 + 5^2 = 13^2$

$x^2 = 169 - 25 = 144$

$x = 12$



$x^2 + 3^2 = 5^2$

$x^2 = 25 - 9 = 16$

$x = 4$

In quadrant II, sin is positive and cosine is negative, so

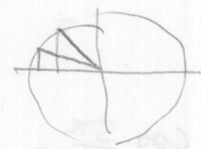
$\cos u = \frac{-12}{13}$ and $\sin v = \frac{4}{5}$.

So $\sin(u+v) = \sin u \cos v + \cos u \sin v$

$= 5/13 \cdot -3/5 + -12/13 \cdot \frac{4}{5} = \frac{-15}{65} - \frac{48}{65} = \boxed{-\frac{63}{65}}$

Evens.

8. a) $\sin\left(\frac{3\pi}{4} + \frac{5\pi}{6}\right) = \sin\frac{3\pi}{4}\cos\frac{5\pi}{6} + \sin\frac{5\pi}{6}\cos\frac{3\pi}{4}$

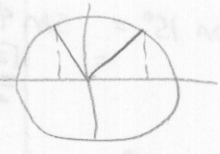


$= \frac{\sqrt{2}}{2} \cdot -\frac{\sqrt{3}}{2} + \frac{1}{2} \cdot -\frac{\sqrt{2}}{2}$

$= \boxed{\frac{-\sqrt{6} - \sqrt{2}}{4}}$

b) $\sin\frac{3\pi}{4} + \sin\frac{5\pi}{6} = \frac{\sqrt{2}}{2} + \frac{1}{2} = \boxed{\frac{\sqrt{2}+1}{2}}$

10. a) $\cos(120° + 45°) = \cos 120° \cos 45° - \sin 120° \sin 45°$

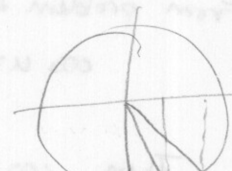


$= -\frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

$= \boxed{\frac{-\sqrt{2} - \sqrt{6}}{4}}$

b) $\cos(120°) + \cos 45° = -\frac{1}{2} + \frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{2} - 1}{2}}$

12. a) $\sin(315° - 60°) = \sin 315° \cos(-60°) + \sin(-60°)\cos(315°)$

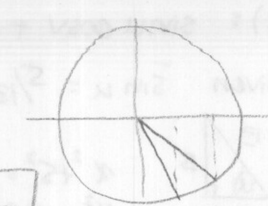


$= -\frac{\sqrt{2}}{2} \cdot \frac{1}{2} + -\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

$= \boxed{\frac{-\sqrt{2} - \sqrt{6}}{4}}$

b) $\sin(315°) - \sin 60° = -\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} = \boxed{\frac{\sqrt{3} - \sqrt{2}}{2}}$

22. $\theta = -\frac{7\pi}{12} = -\frac{\pi}{3} - \frac{\pi}{4}$.



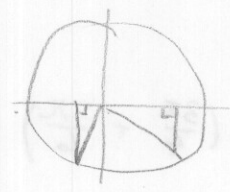
$$\sin -\frac{7\pi}{12} = \sin\left(-\frac{\pi}{3}\right)\cos\left(-\frac{\pi}{4}\right) + \sin\left(-\frac{\pi}{4}\right)\cos\left(-\frac{\pi}{3}\right)$$

$$= -\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + -\frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \boxed{\frac{-\sqrt{6}-\sqrt{2}}{4}}$$

$$\cos -\frac{7\pi}{12} = \cos\left(-\frac{\pi}{3}\right)\cos\left(-\frac{\pi}{4}\right) - \sin\left(-\frac{\pi}{3}\right)\sin\left(-\frac{\pi}{4}\right)$$

$$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} - -\frac{\sqrt{3}}{2}\cdot -\frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$$

24. $\theta = \frac{5\pi}{12} = \frac{8\pi}{12} - \frac{3\pi}{12} = \frac{4\pi}{3} - \frac{\pi}{4}$.



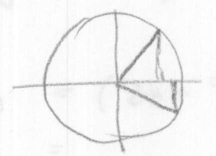
$$\sin\frac{5\pi}{12} = \sin\frac{4\pi}{3}\cos\left(-\frac{\pi}{4}\right) + \sin\left(-\frac{\pi}{4}\right)\cos\left(\frac{4\pi}{3}\right)$$

$$= -\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + -\frac{\sqrt{2}}{2}\cdot -\frac{1}{2} = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$$

$$\cos\frac{5\pi}{12} = \cos\frac{4\pi}{3}\cos\left(-\frac{\pi}{4}\right) - \sin\left(\frac{4\pi}{3}\right)\sin\left(-\frac{\pi}{4}\right)$$

$$= -\tfrac{1}{2}\cdot\frac{\sqrt{2}}{2} - -\frac{\sqrt{3}}{2}\cdot -\frac{\sqrt{2}}{2} = \boxed{\frac{-\sqrt{2}-\sqrt{6}}{4}}$$

28. $\theta = 15° = 45° - 30°$



$$\sin 15° = \sin 45° \cos(-30°) + \sin(-30°)\cos 45°$$

$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + -\frac{1}{2}\cdot\frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$$

$$\cos 15° = \cos 45° \cos(-30°) - \sin 45° \sin(-30°)$$

$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot -\frac{1}{2} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$$

44. From problem 43, if $\sin u = 5/13$ and $\cos v = -3/5$, u, v both in quadrant II,
$\cos u = -\frac{12}{13}$ and $\sin v = 4/5$

Then $\cos(u-v) = \cos u \cos(-v) - \sin u \sin(-v)$

$= \cos u \cos v + \sin u \sin v$ (since cosine is an even function and sine is an odd function)

$$= \frac{-12}{13}\cdot\frac{-3}{5} + \frac{5}{13}\cdot\frac{4}{5}$$

$$= \frac{36+20}{65} = \boxed{\frac{56}{65}}$$

46. From problem 43, $\sin u = 5/13$ and $\cos v = -3/5$, u, v in quadrant II implies $\cos u = -12/13$ and $\sin v = 4/5$.

$$\sin(v-u) = \sin v \cos(-u) + \sin(-u)\cos v$$
$$= \sin v \cos u - \sin u \cos v$$ since sine is an odd function and cosine is an even function.
$$= \frac{4}{5}\cdot\frac{-12}{13} - \frac{5}{13}\cdot\frac{-3}{5}$$
$$= \frac{-48-15}{65} = \boxed{\frac{-63}{65}}$$

96 a) $\sin(u+v) \neq \sin u + \sin v$

For example, let $u = \frac{\pi}{2}$, $v = \frac{\pi}{2}$

Then $\sin(u+v) = \sin(\pi) = 0$, but $\sin\frac{\pi}{2} + \sin\frac{\pi}{2} = 1+1 = 2$.

b) $\sin(u-v) \neq \sin u - \sin v$

For example, let $u = \pi$ and $v = \frac{\pi}{2}$.

Then $\sin(u-v) = \sin\frac{\pi}{2} = 1$, but $\sin u - \sin v = \sin\pi - \sin\frac{\pi}{2} = 0-1 = -1$.

c) $\cos(u+v) \neq \cos u + \cos v$

For example, let $u = \frac{\pi}{2}$, $v = \frac{\pi}{2}$

Then $\cos(u+v) = \cos(\pi) = -1$, but $\cos\frac{\pi}{2} + \cos\frac{\pi}{2} = 0+0 = 0$.

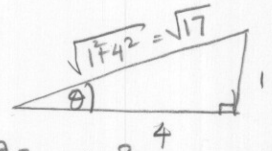
d) $\cos(u-v) \neq \cos u - \cos v$

For example, let $u = \frac{\pi}{2}$, $v = 0$

Then $\cos(u-v) = \cos\frac{\pi}{2} = 0$, but $\cos\frac{\pi}{2} - \cos 0 = 0 - 1 = -1$.

Section 10.5

Odds

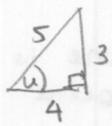


11. $\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(\frac{4}{\sqrt{17}}\right)^2 - \left(\frac{1}{\sqrt{17}}\right)^2 = \frac{16-1}{17} = \boxed{\frac{15}{17}}$

17. $\sin 4\theta = \sin(2(2\theta)) = 2\sin 2\theta \cos 2\theta$
$$= 2(2\sin\theta\cos\theta)(\cos^2\theta - \sin^2\theta)$$
$$= 4\cdot\frac{1}{\sqrt{17}}\cdot\frac{4}{\sqrt{17}}\left(\frac{15}{17}\right) = \frac{16\cdot 15}{17^2} = \boxed{\frac{240}{289}}$$

37. $\sin u = -3/5$, $\frac{3\pi}{2} < u < 2\pi$ (i.e. u in quadrant IV).

If $\sin u = -3/5$, $\cos u = 4/5$ (it's positive since cos is positive in quadrant IV).

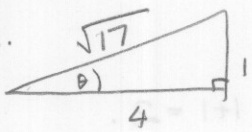


So $\sin 2u = 2\sin u \cos u = 2\cdot\left(-\frac{3}{5}\right)\left(\frac{4}{5}\right) = \boxed{-\frac{24}{25}}$

$\cos 2u = \cos^2 u - \sin^2 u = \frac{16}{25} - \frac{9}{25} = \boxed{\frac{7}{25}}$

Evens

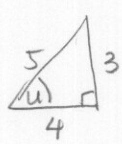
12.



$\sin 2\theta = 2\sin\theta\cos\theta = 2\cdot\frac{1}{\sqrt{17}}\cdot\frac{4}{\sqrt{17}} = \boxed{\frac{8}{17}}$

38. $\cos u = -4/5$, $\frac{\pi}{2} < u < \pi$ (ie u in quadrant II)

If $\cos u = -4/5$, $\sin u = 3/5$ (it's positive since sin is positive in quadrant II)



$\sin 2u = 2\sin u\cos u = 2\cdot\frac{3}{5}\cdot\frac{-4}{5} = \boxed{\frac{-24}{25}}$

$\cos 2u = \cos^2 u - \sin^2 u = \frac{16}{25} - \frac{9}{25} = \boxed{\frac{7}{25}}$