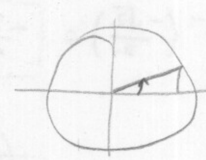
Homework 8 Solutions

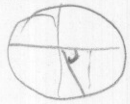
Section 9.7

odds:

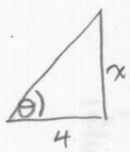
5. $\arcsin\frac{1}{2} = \boxed{\frac{\pi}{6}}$ since $\sin\frac{\pi}{6} = \frac{1}{2}$ and $-\pi/2 \le \frac{\pi}{6} \le \frac{\pi}{2}$



17. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \boxed{-\frac{\pi}{3}}$ since $\sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$ and $-\frac{\pi}{2} \le \frac{-\pi}{3} \le \frac{\pi}{2}$

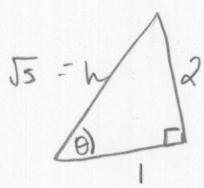


43.



$\tan\theta = \frac{x}{4}$, so $\boxed{\theta = \arctan\frac{x}{4}}$

57. $\cos(\tan^{-1} 2)$



If $\tan^{-1} 2 = \theta$, we get the triangle to the left

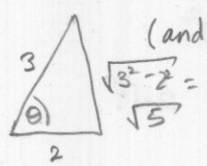
$h^2 = 1^2 + 2^2$, so $h = \sqrt{5}$

Then $\cos(\theta) = \frac{1}{\sqrt{5}} = \boxed{\frac{\sqrt{5}}{5}}$

63. $\sin\left(\arccos\left(-\frac{2}{3}\right)\right)$

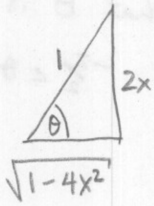
Setting $\theta = \arccos\left(-\frac{2}{3}\right)$, we get the following triangle:

(and θ is in quadrant II, since cos is negative).



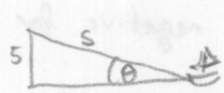
so $\sin\theta = \boxed{\frac{\sqrt{5}}{3}}$ (sin is positive in quadrant II.)

67. $\cos(\arcsin 2x)$



$\cos(\arcsin 2x) = \boxed{\sqrt{1-4x^2}}$

109.



a) $\frac{5}{s} = \sin\theta$, so $\boxed{\theta = \arcsin\frac{5}{s}}$

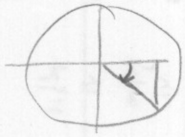
b) $\theta = \arcsin\frac{5}{40} \approx$

$\theta = \arcsin\frac{5}{20} \approx$

Evens

6. $\arcsin 0 = \boxed{0}$ since $\sin 0 = 0$ and $-\frac{\pi}{2} \le 0 \le \frac{\pi}{2}$

12. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \boxed{-\frac{\pi}{4}}$ since $\sin\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$ and $-\frac{\pi}{2} \le -\frac{\pi}{4} \le \frac{\pi}{2}$

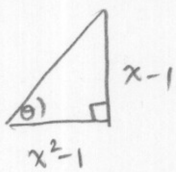


18. $\tan^{-1}\left(\frac{-\sqrt{3}}{3}\right) = \tan^{-1}\left(-\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) = \boxed{-\frac{\pi}{6}}$

At $-\frac{\pi}{6}$, sin is $-\frac{1}{2}$ and cos is $\frac{\sqrt{3}}{2}$, So tan is $-\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$.

Also. $-\frac{\pi}{2} < -\frac{\pi}{6} < \frac{\pi}{2}$.

48.

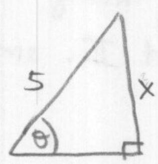


$\tan\theta = \frac{x-1}{x^2-1} = \frac{\cancel{x-1}}{(x+1)\cancel{(x-1)}} = \frac{1}{x+1}$

So $\boxed{\theta = \arctan \frac{1}{x+1}}$

58. $\sin\left(\cos^{-1}\frac{\sqrt{5}}{5}\right)$

If $\theta = \cos^{-1}\frac{\sqrt{5}}{5}$, we get this triangle



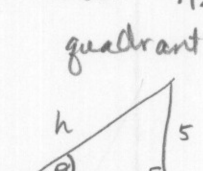
$5^2 = x^2 + (\sqrt{5})^2$

So $x = \sqrt{20} = 2\sqrt{5}$.

Then $\sin\theta = \boxed{\frac{2\sqrt{5}}{5}}$

60. $\csc(\arctan(-5/12))$

If $\arctan -5/12 = \theta$, we get this triangle. Note that θ is in quadrant IV since $\tan\theta$ is negative (and $-\pi/2 < \theta < \frac{\pi}{2}$).



$h^2 = 5^2 + 12^2 = 169$

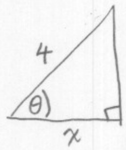
So $h = 13$.

$\csc\theta = \pm\frac{13}{5}$, choose negative since csc is negative for angles in quadrant IV

$\csc\theta = \boxed{-\frac{13}{5}}$

62. $\tan\left(\arcsin\left(-\frac{3}{4}\right)\right)$

If $\arcsin(-3/4) = \theta$, we get the following triangle. Note that θ is in quadrant IV since $\sin\theta$ is negative and $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$.

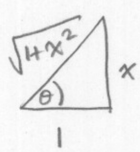


$x^2 + 3^2 = 4^2$, so $x = \pm\sqrt{7}$

Then $\tan\theta = \frac{\pm 3}{\sqrt{7}}$. Choose negative since θ is in quadrant IV and tan is negative in quadrant IV.

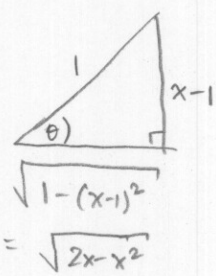
$\tan\theta = \frac{-3}{\sqrt{7}} = \boxed{\frac{-3\sqrt{7}}{7}}$

66. $\sin(\arctan x)$



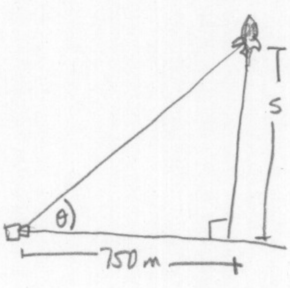
$\boxed{\sin\theta = \frac{x}{\sqrt{1+x^2}}}$

70. $\sec(\arcsin(x-1))$



$\sec\theta = \frac{1}{\cos\theta} = \boxed{\frac{1}{\sqrt{2x-x^2}}}$

110.



a) $\frac{s}{750} = \tan\theta$, so $\boxed{\theta = \arctan\frac{s}{750}}$

b) $s = 300$ m:

$\theta = \arctan\frac{300}{750} \approx$

$s = 1200$ m

$\theta = \arctan\frac{1200}{750} \approx$

Section 10.1

Odds

11. $\sin x = \frac{1}{2}, \quad \cos x = \frac{\sqrt{3}}{2}.$

$\tan x = \frac{\sin x}{\cos x} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \qquad \cot x = \frac{\cos x}{\sin x} = \sqrt{3}$

$\sec x = \frac{1}{\cos x} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3} \qquad \csc x = \frac{1}{\sin x} = 2.$

39. $\sec\alpha \cdot \frac{\sin\alpha}{\tan\alpha} = \frac{1}{\cos\alpha} \cdot \frac{\sin\alpha}{\left(\frac{\sin\alpha}{\cos\alpha}\right)} = \frac{1}{\cancel{\cos\alpha}} \cdot \sin\alpha \cdot \frac{\cancel{\cos\alpha}}{\cancel{\sin\alpha}} = \boxed{1}$

53. $\frac{\sec^2 x - 1}{\sec x - 1} = \frac{(\sec x + 1)\cancel{(\sec x - 1)}}{\cancel{\sec x - 1}} = \boxed{\sec x + 1}$

Evens

12. $\tan x = \frac{\sqrt{3}}{3}, \quad \cos x = -\frac{\sqrt{3}}{2}.$

Since $\tan x = \frac{\sin x}{\cos x}$, $\sin x = (\cos x)(\tan x) = \frac{-1}{2}.$

$\sec x = \frac{1}{\cos x} = \frac{-2}{\sqrt{3}} = \frac{-2\sqrt{3}}{3}$

$\cot x = \frac{1}{\tan x} = \frac{3}{\sqrt{3}} = \sqrt{3}$

$\csc x = \frac{1}{\sin x} = -2.$

16. $\cot\varphi = -3, \quad \sin\varphi = \frac{\sqrt{10}}{10}.$

$\cot\varphi = \frac{\cos\varphi}{\sin\varphi}$, so $\cos\varphi = (\sin\varphi)(\cot\varphi) = \frac{-3\sqrt{10}}{10}$

$\tan\varphi = \frac{1}{\cot\varphi} = -\frac{1}{3}$

$\sec\varphi = \frac{1}{\cos\varphi} = \frac{-10}{3\sqrt{10}} = \frac{-\sqrt{10}}{3}.$

$\csc\varphi = \frac{1}{\sin\varphi} = \frac{10}{\sqrt{10}} = \sqrt{10}.$

24. $\tan\theta$ is undefined, $\sin\theta > 0.$

$\tan\theta$ is undefined at $\frac{\pi}{2}$ and $\frac{3\pi}{2}$, but sin is positive at $\frac{\pi}{2}$ and negative at $\frac{3\pi}{2}$.

So $\theta = \frac{\pi}{2}$ (or $\frac{\pi}{2} +$ a multiple of 2π).

and $\sin\theta = 1, \quad \cos\theta = 0$

$\tan\theta$ undefined. $\sec\theta = \frac{1}{\cos\theta}$ is undefined also

$\cot\theta = \frac{\cos\theta}{\sin\theta} = 0$ and $\csc\theta = \frac{1}{\sin\theta} = 1.$

38. $\dfrac{1}{\tan^2 x + 1} = \dfrac{1}{\sec^2 x} = \boxed{\cos^2 x}$

56. $1 - 2\cos^2 x + \cos^4 x = (1 - \cos^2 x)^2 = (\sin^2 x)^2 = \boxed{\sin^4 x}$

64. $(3 - 3\sin x)(3 + 3\sin x) = 9 - 9\sin^2 x = 9(1 - \sin^2 x) = \boxed{9\cos^2 x}$