Homework 4 Solutions

Section 8.3, odds

9. $\int \frac{x^4-4}{x}\,dx = \int \frac{x^4}{x} - \frac{4}{x}\,dx$

$= \int x^3 dx - 4\int \frac{1}{x}\,dx = \boxed{\frac{x^4}{4} - 4\ln|x| + C}$

19. $\int \frac{(\ln x)^2}{x}\,dx$

Use substitution $u = \ln x$
Then $du = \frac{1}{x}\,dx$

$\int u^2\,du = \frac{u^3}{3} + C = \boxed{\frac{1}{3}(\ln x)^3 + C}$

29. $\int \frac{1}{1+\sqrt{2x}}\,dx$

Use substitution $u = 1+\sqrt{2x} \longrightarrow u-1 = \sqrt{2x}$

$du = \frac{\sqrt{2}}{2} x^{-\frac{1}{2}}\,dx$

$= \frac{1}{\sqrt{2x}}\,dx \longrightarrow \sqrt{2x}\,du = dx$

$(u-1)\,du = dx$

$\int \frac{(u-1)}{u}\,du = \int 1 - \frac{1}{u}\,du$

$= u - \ln|u| + C$

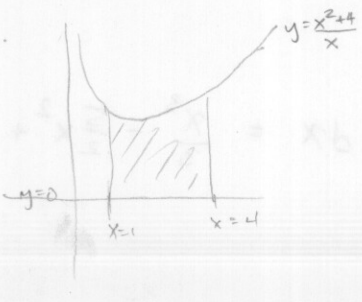
$= \boxed{1+\sqrt{2x} - \ln|1+\sqrt{2x}| + C}$

33. $\int 3^x\,dx = \int (e^{\ln 3})^x\,dx = \int e^{(\ln 3)\cdot x}\,dx$

$= \frac{1}{\ln 3} e^{(\ln 3)x} + C$

$= \boxed{\frac{1}{\ln 3} 3^x + C}$

59.



$\int_1^4 \frac{x^2+4}{x}\,dx = \int_1^4 x + \frac{4}{x}\,dx$

$= \frac{x^2}{2}\Big|_1^4 + 4\ln|x|\Big|_1^4 = \left(8-\frac{1}{2}\right) + (4\ln 4 - 4\ln 1)$

$= \boxed{\frac{15}{2} + 4\ln 4}$

75. $\frac{dP}{dt} = \frac{3000}{1+0.25t}$

$P = \int \frac{3000}{1+0.25t}\, dt$

Use substitution: $u = 1+0.25t$
$du = 0.25\, dt$
So $4\, du = dt$

$P = 4\int \frac{3000}{u}\, du = 12{,}000 \ln|u| + C$

$= 12{,}000 \ln|1+0.25t| + C$

$P(0) = 1000 = 12{,}000 \ln(1) + C$
$= C$

So $\boxed{P(t) = 12{,}000 \ln|1+0.25t| + 1{,}000}$

$P(3) = 12000 \ln(1.75) + 1000 \approx \boxed{7715}$

Evens:

8. $\int \frac{x^2}{3-x^3}\, dx$

Use substitution: $u = 3-x^3$
$du = -3x^2\, dx$, so $-\frac{1}{3}\, du = x^2\, dx$

$\int \frac{1}{u}\, du = \ln|u| + C = \boxed{\ln|3-x^3| + C}$

16. $\int \frac{x^3-6x-20}{x+5}\, dx$

Use polynomial long division first

$$
\begin{array}{r}
x^2 - 5x + 19 \\
x+5 \enclose{longdiv}{x^3 + 0x^2 - 6x - 20} \\
-(x^3 + 5x^2) \\
-5x^2 - 6x - 20 \\
-(-5x^2 - 25x) \\
19x - 20 \\
-(19x + 95) \\
-115
\end{array}
$$

So the integral becomes $\int x^2 - 5x + 19 - \frac{115}{x+5}\, dx = \frac{x^3}{3} - \frac{5}{2}x^2 + 19x - 115\int \frac{1}{x+5}dx$

16 cont.

For $\int \frac{1}{x+5}\,dx$, use substitution.

$u = x+5$

$du = dx$

So $\int \frac{1}{x+5}\,dx = \int \frac{1}{u}\,du = \ln|u| + C = \ln|x+5| + C$

Therefore, the original integral is equal to

$$\boxed{\frac{x^3}{3} - \frac{5}{2}x^2 + 19x - 115\ln|x+5| + C}$$

20. $\int \frac{1}{x\ln(x^3)}\,dx = \int \frac{1}{3x\ln x}\,dx$ since $\ln(x^3) = 3\ln x$

Use substitution: $u = \ln x$. Then $du = \frac{1}{x}\,dx$.

So we get $\frac{1}{3}\int \frac{1}{u}\,du = \frac{1}{3}\ln|u| + C$

$= \boxed{\frac{1}{3}\ln|\ln x| + C}$

26. $\int \frac{1}{x^{2/3}(1+x^{1/3})}\,dx$

Use substitution: $u = 1 + x^{1/3}$.

$du = \frac{1}{3}x^{-2/3}\,dx = \frac{1}{3x^{2/3}}\,dx$, so $3\,du = \frac{1}{x^{2/3}}\,dx$.

So we get $3\int \frac{1}{u}\,du = 3\ln|u| + C = \boxed{3\ln|1+x^{1/3}| + C}$

34. $\int 5^{-x}\,dx = \int (e^{\ln 5})^{-x}\,dx = \int e^{(-\ln 5)\cdot x}\,dx$

$= \frac{1}{-\ln 5}\,e^{(-\ln 5)x} + C$

$= \boxed{\frac{-1}{\ln 5}\,5^{-x} + C}$

44. $\int_e^{e^2} \frac{1}{x\ln x}\,dx$ Use substitution: $u = \ln x$, $du = \frac{1}{x}\,dx$. Change limits: If $x = e$, $u = 1$. If $x = e^2$, $u = 2$

$\int_1^2 \frac{1}{u}\,du = \ln|u|\Big|_1^2 = \ln 2 - \ln 1 = \boxed{\ln 2}$

46. $\int_0^1 \frac{x-1}{x+1}\,dx = \int_0^1 \frac{(x+1)-2}{x+1}\,dx = \int_0^1 1 - \frac{2}{x+1}\,dx$

$$= \int_0^1 dx - 2\int_0^1 \frac{1}{x+1}\,dx$$

$$= x\Big|_0^1 - 2\int_0^1 \frac{1}{x+1}\,dx$$

$$= (1-0) - 2\int_0^1 \frac{1}{x+1}\,dx$$

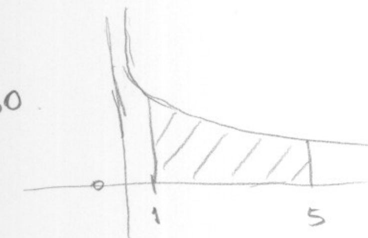
Use substitution: $u = x+1$ Then $du = dx$.
If $x=0$, $u=1$; If $x=1$, $u=2$.

Get $1 - 2\int_1^2 \frac{1}{u}\,du = 1 - 2\ln|u|\Big|_1^2$

$$= 1 - (2\ln 2 - 2\ln 1)$$

$$= \boxed{1 - 2\ln 2}$$

60.



$\int_1^5 \frac{x+6}{x}\,dx = \int_1^5 1 + \frac{6}{x}\,dx$

$$= \int_1^5 dx + 6\int_1^5 \frac{1}{x}\,dx$$

$$= x\Big|_1^5 + 6\ln|x|\Big|_1^5$$

$$= (5-1) + (6\ln 5 - 6\ln 1)$$

$$= \boxed{4 + 6\ln 5}$$

Section 8.4 odds.

7. $y' = \sqrt{x}\,y$

$$\frac{dy}{dx} = \sqrt{x}\,y$$

$$\frac{dy}{y} = \sqrt{x}\,dx$$

$$\int \frac{dy}{y} = \int \sqrt{x}\,dx$$

$$\ln|y| = \frac{2}{3}x^{3/2} + C_1$$

$$\boxed{y = Ce^{\frac{2}{3}x^{3/2}}}$$

17. $\frac{dy}{dt} = \frac{1}{2}t$

$y = \int \frac{1}{2} t \, dt = \frac{t^2}{4} + c$

$y(0) = 10 = \frac{1}{4}0^2 + C = C$

So $\boxed{y = \frac{1}{4}t^2 + 10}$

Evens.

8. $y' = x(1+y)$

$\frac{dy}{dx} = x(1+y)$

$\frac{dy}{1+y} = x\,dx$

$\int \frac{dy}{1+y} = \int x\,dx$

$\ln|1+y| = \frac{x^2}{2} + c_1$

So $1+y = Ce^{x^2/2}$

$\boxed{y = Ce^{x^2/2} - 1}$

10. $xy + y' = 100x$

$\frac{dy}{dx} = 100x - xy = x(100-y)$

$\frac{dy}{100-y} = x\,dx$

$\int \frac{1}{100-y}\,dy = \int x\,dx$

$u = 100 - y$

$du = -dy$

$-\int \frac{1}{u}\,du = \frac{x^2}{2} + c_1$

$-\ln|100-y| = \frac{x^2}{2} + c_1$

$\ln|100-y| = -\frac{x^2}{2} + c_2$

$(c_2 = -c_1)$

So

$100 - y = e^{-x^2/2} \cdot C$

$\boxed{y = 100 - Ce^{-x^2/2}}$

12. $\frac{dP}{dt} = k(25-t)$

$P = \int k(25-t)\,dt = \boxed{25kt - \frac{kt^2}{2} + C}$

22. $\frac{dN}{dt} = kN.$ Given $N(0) = 250$, $N(1) = 400$.

$\frac{dN}{N} = k\,dt$

$\int \frac{dN}{N} = \int k\,dt$

$\ln|N| = kt + C_1$

$N = Ce^{kt}$

$N(0) = 250 = Ce^0 = C \implies C = 250$

$N(1) = 400 = 250e^k \longrightarrow e^k = \frac{400}{250} = \frac{8}{5}$

$k = \ln(\frac{8}{5})$

So $N(t) = 250\,e^{\ln\frac{8}{5}t}$

and $N(4) = 250\,e^{(\ln\frac{8}{5})(4)} = 250 \cdot (\frac{8}{5})^4 = \boxed{1638.4}$

48. Number of bacteria increasing by exponential growth means

$P(t) = Ce^{kt}$

$P(2) = Ce^{2k} = 125$

$P(4) = Ce^{4k} = 350$

$\frac{P(4)}{P(2)} = \frac{Ce^{4k}}{Ce^{2k}} = \frac{350}{125} = \frac{14}{5}$

$\frac{Ce^{4k}}{Ce^{2k}} = e^{2k}$

$2k = \ln\frac{14}{5} \implies k = \frac{1}{2}\ln\frac{14}{5}$

$P(2) = Ce^{\ln\frac{14}{5}} = 125 \implies \frac{14}{5}C = 125 \implies C = \frac{625}{14}$

a) Initial population is $\boxed{\frac{625}{14}}$

48 b) $\boxed{P(t) = \frac{625}{14} e^{\frac{t}{2}\ln\frac{14}{5}}}$

c) $P(8) = \frac{625}{14} e^{4\ln\frac{14}{5}} = \frac{625}{14} e^{4\ln\frac{14}{5}} = \frac{625}{14}\cdot\left(\frac{14}{5}\right)^4 =$

d) $P(t) = \frac{625}{14} e^{\frac{t}{2}\ln\frac{14}{5}} = 25{,}000$

$$e^{\frac{t}{2}\ln\frac{14}{5}} = 560$$

$$\frac{t}{2}\ln\frac{14}{5} = \ln 560$$

$$t = \frac{2\ln 560}{\ln\frac{14}{5}} \approx \boxed{12.3}$$

54. $R = \frac{\ln I - \ln I_0}{\ln 10}$ (Given:) Assume $I_0 = 1$, so $\ln I_0 = 0$.

So $R = \frac{\ln I}{\ln 10}$

a) $8.3 = \frac{\ln I}{\ln 10} \Rightarrow I = e^{8.3\ln 10} = \boxed{10^{8.3}}$ $(\approx 19.95\times 10^7)$.

b) Suppose $R = \frac{\ln I}{\ln 10}$.

$$2R = \frac{2\ln I}{\ln 10} = \frac{\ln I^2}{\ln 10}.$$

So if the Richter scale measure doubles, the intensity gets $\boxed{\text{squared}}$.

c) $\boxed{\frac{dR}{dI} = \frac{1}{I\ln 10}}$