Homework 2 Solutions

Section 6.4

9. $\int_{-1}^{1} (t^2 - 2)\, dt = \left(\frac{t^3}{3} - 2t\right)\Big|_{-1}^{1} = \left(\frac{1}{3} - 2\right) - \left(\frac{-1}{3} + 2\right) = \frac{2}{3} - 4 = \boxed{\frac{-10}{3}}$

15. $\int_1^4 \frac{u-2}{\sqrt{u}}\, du = \int_1^4 \sqrt{u} - \frac{2}{\sqrt{u}}\, du = \left(\frac{u^{3/2}}{3/2} - \frac{2u^{1/2}}{1/2}\right)\Big|_1^4 = \left(\frac{2}{3}\cdot 8 - 8\right) - \left(\frac{2}{3} - 4\right)$

$= \frac{-10}{3} + 4 = \boxed{\frac{2}{3}}$

19. $\int_0^1 \frac{x - \sqrt{x}}{3}\, dx = \frac{1}{3}\int_0^1 x - \sqrt{x}\, dx = \frac{1}{3}\left(\left[\frac{x^2}{2} - \frac{x^{3/2}}{3/2}\right]_0^1\right)$

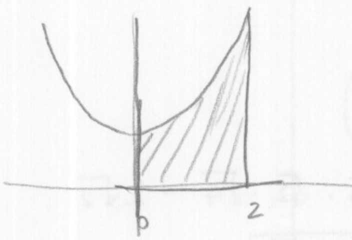
$= \frac{1}{3}\left[\left(\frac{1}{2} - \frac{2}{3}\right) - \left(\frac{0}{2} - \frac{0}{3/2}\right)\right] = \frac{1}{3}\cdot\frac{-1}{6} = \boxed{\frac{-1}{18}}$

27. $y = x - x^2$

Area $= \int_0^1 x - x^2\, dx = \left(\frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_0^1 = \left(\frac{1}{2} - \frac{1}{3}\right) - (0 - 0)$

$= \boxed{\frac{1}{6}}$

33. $y = 5x^2 + 2$, $x = 0$, $x = 2$, $y = 0$.



$\int_0^2 5x^2 + 2\, dx = \left(5\frac{x^3}{3} + 2x\right)\Big|_0^2 = \frac{40}{3} + 4 - (0 + 0)$

$= \boxed{\frac{52}{3}}$

67. a) $F(x) = \int_8^x \sqrt[3]{t}\, dt = \int_8^x t^{1/3}\, dt = \frac{t^{4/3}}{4/3}\Big|_8^x = \boxed{\frac{3}{4}x^{4/3} - 12}$

b) $\frac{d}{dx}F(x) = \frac{d}{dx}\left(\frac{3}{4}x^{4/3} - 12\right) = x^{1/3} - 0 = \sqrt[3]{x}$ ✓

89. $v(t) = \frac{1}{\sqrt{t}}$ $\quad t > 0$

At $t = 1$, position is $x = 4$

Find total distance traveled from $t = 1$ to $t = 4$.

Distance $= \int_1^4 \left|\frac{1}{\sqrt{t}}\right| dt = \int_1^4 \frac{1}{\sqrt{t}}\, dt = \frac{t^{1/2}}{1/2}\Big|_1^4 = 4 - 2 = \boxed{2}$

8. $\int_2^5 (-3v+4)\,dv = \left(\frac{-3v^2}{2} + 4v\right)\Big|_2^5 = \left(\frac{-75}{2} + 20\right) - (-6+8)$

$= \boxed{-\frac{39}{2}}$

10. $\int_1^7 (6x^2+2x-3)\,dx = \left(\frac{6x^3}{3} + \frac{2x^2}{2} - 3x\right)\Big|_1^7 = \left(2x^3 + x^2 - 3x\right)\Big|_1^7$

$= (686+49-21) - (2+1-3)$

$= \boxed{714}$

12. $\int_{-1}^1 (t^3 - 9t)\,dt = \left(\frac{t^4}{4} - \frac{9t^2}{2}\right)\Big|_{-1}^1 = \left(\frac{1}{4} - \frac{9}{2}\right) - \left(\frac{1}{4} - \frac{9}{2}\right) = \boxed{0}$

14. $\int_{-2}^{-1} \left(u - \frac{1}{u^2}\right)du = \left(\frac{u^2}{2} - \frac{u^{-1}}{-1}\right)\Big|_{-2}^{-1} = \left(\frac{1}{2} - 1\right) - \left(\frac{4}{2} + \frac{-1}{2}\right)$

$= \boxed{-2}$

16. $\int_{-3}^3 v^{1/3}\,dv = \frac{v^{4/3}}{4/3}\Big|_{-3}^3 = \frac{3}{4}\left(3^{4/3} - (-3)^{4/3}\right)$

$= \frac{3}{4}\left(\sqrt[3]{3^4} - \sqrt[3]{(-3)^4}\right)$

$= \frac{3}{4}\left(\sqrt[3]{3^4} - \sqrt[3]{3^4}\right) = \boxed{0}$

18. $\int_1^8 \sqrt{\frac{2}{x}}\,dx = \int_1^8 \frac{\sqrt{2}}{\sqrt{x}}\,dx = \sqrt{2}\int_1^8 x^{-1/2}\,dx = \sqrt{2}\left(\frac{x^{1/2}}{1/2}\Big|_1^8\right)$

$= 2\sqrt{2}\cdot\sqrt{8} - 2\sqrt{2}\cdot 1 = 2\sqrt{2}\cdot\sqrt{2}\cdot\sqrt{4} - 2\sqrt{2}$

$= \boxed{8 - 2\sqrt{2}}$

20. $\int_0^2 (2-t)\sqrt{t}\,dt = \int_0^2 2\sqrt{t} - t^{3/2}\,dt = \left(\frac{2t^{3/2}}{3/2} - \frac{t^{5/2}}{5/2}\right)\Big|_0^2$

$= \left(\frac{4}{3}\sqrt{8} - \frac{2}{5}\sqrt{32}\right) - (0-0)$

$= \frac{8\sqrt{2}}{3} - \frac{8\sqrt{2}}{5} = \boxed{\frac{16\sqrt{2}}{15}}$

28. $y = -x^2+2x+3$

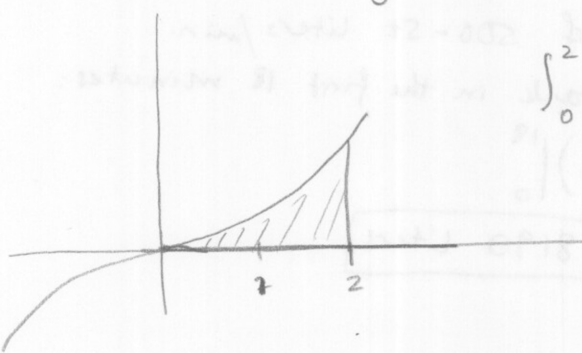
$$\int_{-1}^{3} -x^2+2x+3 \, dx = \left(-\frac{x^3}{3} + \frac{2x^2}{2} + 3x\right)\Big|_{-1}^{3} = \left(-\frac{27}{3}+9+9\right) - \left(\frac{1}{3}+1-3\right)$$

$$= 9 - \left(-\frac{5}{3}\right) = \boxed{\frac{32}{3}}$$

30. $y = \frac{1}{x^2}$

$$\int_1^2 \frac{1}{x^2}\, dx = \int_1^2 x^{-2}\, dx = \frac{x^{-1}}{-1}\Big|_1^2 = \frac{-1}{2} - \frac{-1}{1} = \frac{-1}{2} + 1 = \boxed{\frac{1}{2}}$$

34. $y = x^3+x, \quad x=2, \quad y=0.$



$$\int_0^2 x^3+x\, dx = \left(\frac{x^4}{4} + \frac{x^2}{2}\right)\Big|_0^2 = (4+2) - (0+0) = \boxed{6}$$

36. $y = (3-x)\sqrt{x}, \; y=0$

$$\int_0^3 (3-x)\sqrt{x}\, dx = 3\int_0^3 x^{1/2}\, dx - \int_0^3 x^{3/2}\, dx$$

$$= 3\left(\frac{x^{3/2}}{3/2}\Big|_0^3\right) - \left(\frac{x^{5/2}}{5/2}\right)\Big|_0^3$$

$$= \left(2\cdot 3^{3/2} - 0\right) - \left(\frac{2}{5}\cdot 3^{5/2} - 0\right)$$

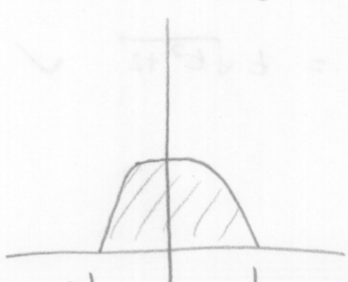
$$= 6\sqrt{3} - \frac{2}{5}\cdot 9\sqrt{3}$$

$$= \boxed{\frac{12\sqrt{3}}{5}}$$

38. $y = 1-x^4, \quad y=0$

$$\int_{-1}^{1} 1-x^4\, dx = \left(x - \frac{x^5}{5}\right)\Big|_{-1}^{1} = \left(1-\frac{1}{5}\right) - \left(-1+\frac{1}{5}\right)$$

$$= 2 - \frac{2}{5} = \boxed{\frac{8}{5}}$$



68a) $F(x) = \int_4^x \sqrt{t}\, dt = \int_4^x t^{1/2} dt = \left(\frac{t^{3/2}}{3/2}\right)\Big|_4^x = \frac{2}{3}x^{3/2} - \frac{2}{3}\cdot 8$

$= \boxed{\frac{2}{3}x^{3/2} - \frac{16}{3}}$

b) $\frac{d}{dx} F(x) = \frac{d}{dx}\left(\frac{2}{3}x^{3/2} - \frac{16}{3}\right) = x^{1/2} - 0$ ✓

72. $F(x) = \int_1^x \frac{t^2}{t^2+1}\, dt$

$\boxed{F'(x) = \frac{x^2}{x^2+1}}$

90. Water flows out of a tank at a rate of $500-5t$ liters/min.

Amount of water that flows out of the tank in the first 18 minutes:

$\int_0^{18} 500 - 5t\, dt = \left(500t - 5\frac{t^2}{2}\right)\Big|_0^{18}$

$= 8190 - 0 = \boxed{8190 \text{ liters}}$

Section 6.5

1. $\int (8x^2+1)^2 (16x)\, dx$

$u = g(x) = 8x^2+1$

$du = g'(x)dx = 16x\, dx$

So $\int (8x^2+1)^2(16x)\, dx = \int u^2\, du = \frac{u^3}{3} + C = \frac{(8x^2+1)^3}{3} + C$

15. $\int t\sqrt{t^2+2}\, dt = ?$

$u = t^2+2 \quad du = 2t\, dt$, so $\frac{1}{2}du = t\,dt$

Substitute to get:

$\frac{1}{2}\int \sqrt{u}\, du = \frac{\frac{1}{2}u^{3/2}}{3/2} + C = \frac{1}{3}u^{3/2} + C = \boxed{\frac{1}{3}(t^2+2)^{3/2} + C}$

Check: $\frac{d}{dt}\left(\frac{1}{3}(t^2+2)^{3/2} + C\right) = \frac{1}{2}(t^2+2)^{1/2}\cdot 2t = t\sqrt{t^2+2}$ ✓

19. $\int \frac{x}{(1-x^2)^3}\,dx = ?$

$u = 1-x^2 \qquad du = -2x\,dx$ so $-\frac{1}{2}\,du = x\,dx$.

$-\frac{1}{2}\int \frac{1}{u^3}\,du = -\frac{1}{2}\,\frac{u^{-2}}{-2} + C = \frac{1}{4u^2} + C = \boxed{\frac{1}{4(1-x^2)^2} + C}$

Check. $\frac{d}{dx}\left(\frac{1}{4(1-x^2)^2} + C\right) = \frac{-2}{4}(1-x^2)^{-3}\cdot(-2x) + 0$

$= \frac{x}{(1-x^2)^3}$ ✓

4. $\int \frac{x}{\sqrt{x^2+1}}\,dx = ?$

$u = x^2+1$
$du = 2x\,dx$

so $\int \frac{x}{\sqrt{x^2+1}}\,dx = \int \frac{1}{\sqrt{u}}\cdot\frac{1}{2}\,du = \frac{1}{2}\int u^{-1/2}\,du = \frac{\frac{1}{2}u^{1/2}}{1/2} + C =$

$= u^{1/2} + C = \sqrt{x^2+1} + C$

10. $\int \sqrt[3]{3-4x^2}\,(-8x)\,dx$

$u = 3-4x^2$
$du = -8x\,dx$

$\int \sqrt[3]{u}\,du = \int u^{1/3}\,du = \frac{u^{4/3}}{4/3} + C = \boxed{\frac{3}{4}(3-4x^2)^{4/3} + C}$

12. $\int x^2(x^3+5)^4\,dx$

$u = x^3+5$
$du = 3x^2\,dx \longrightarrow \frac{1}{3}\,du = x^2\,dx$

$\frac{1}{3}\int u^4\,du = \frac{1}{3}\cdot\frac{u^5}{5} + C = \boxed{\frac{1}{15}(x^3+5)^5 + C}$

20. $\int \frac{x^3}{(1+x^4)^2}\,dx = ?$

$u = 1+x^4$
$du = 4x^3\,dx \longrightarrow \frac{1}{4}\,du = x^3\,dx$

$\int \frac{1}{u^2}\,du = \frac{u^{-1}}{-1} + C = \boxed{\frac{-1}{1+x^4} + C}$

26. $\int \left[x^2 + \frac{1}{(3x)^2}\right] dx = \int x^2\, dx + \frac{1}{9}\int \frac{1}{x^2}\, dx$

$$= \frac{x^3}{3} + \frac{1}{9}\,\frac{x^{-1}}{-1} + C$$

$$= \boxed{\frac{x^3}{3} - \frac{1}{9x} + C}$$