Homework 11 Solutions

Section 11.2

Odds.

29. $f(x) = e^{\sin x}$, $(0,1)$.

$f'(x) = e^{\sin x} \cdot \cos x$

$f'(0) = e^{\sin(0)} \cdot \cos(0) = e^0 \cdot 1 = 1$, so the slope of the tangent line is 1.

Tangent line: $(y-1) = 1\cdot(x-0)$

$\boxed{y = x+1}$

41. $y = 4\sec^2 x$

$y' = 4 \cdot 2 \cdot \sec x \cdot \sec x \cdot \tan x$

$= \boxed{8 \sec^2 x \tan x}$

53. $y = \ln|\sin x|$

When $\sin x > 0$, $y = \ln(\sin x)$ and $y' = \frac{1}{\sin x} \cdot \cos x = \frac{\cos x}{\sin x} = \cot x$.

When $\sin x < 0$, $y = \ln(-\sin x)$ and $y' = \frac{-1}{\sin x} \cdot (-\cos x) = \frac{\cos x}{\sin x} = \cot x$.

So $\boxed{y' = \cot x}$

59. $y = \sin(\tan 2x)$

$y' = \cos(\tan 2x) \cdot \sec^2 2x \cdot 2 = \boxed{2\cos(\tan 2x)\sec^2 2x}$

105. A buoy oscillates in simple harmonic motion: $y = A\cos\omega t$.

It moves 3.5 ft (vertically) from the lowest point to the highest point.

It returns to the highest pt every 10 seconds.

a) Amplitude $= \frac{3.5}{2} = \frac{7}{4}$

Period $= \frac{2\pi}{\omega} = 10 \Rightarrow \omega = \frac{\pi}{5}$.

$y = \boxed{\frac{7}{4}\cos\frac{\pi t}{5}}$

b) Velocity $= y' = \left(-\frac{7}{4}\sin\frac{\pi t}{5}\right)\left(\frac{\pi}{5}\right) = \boxed{\frac{-7\pi}{20}\sin\frac{\pi t}{5}}$

Evens

20. $y = \dfrac{\sec x}{x}$

Use the quotient rule: $y' = \dfrac{x \cdot \sec x \tan x - \sec x}{x^2}$

$= \boxed{\dfrac{\sec x \tan x}{x} - \dfrac{\sec x}{x^2}}$

24. $h(\theta) = 5\theta \sec\theta + \theta \tan\theta$

Use the product rule:

$\boxed{h'(\theta) = 5\sec\theta + 5\theta \sec\theta \tan\theta + \tan\theta + \theta \sec^2\theta}$

42. $y = 2\tan^3 x$

Use the chain rule.

$\boxed{y' = 6\tan^2 x \cdot \sec^2 x}$

52. $y = e^x(\sin x + \cos x)$

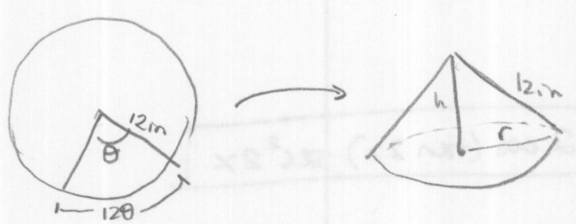
Use the product rule

$y' = e^x(\sin x + \cos x) + e^x(\cos x - \sin x)$

$= e^x \cancel{\sin x} + e^x \cos x + e^x \cos x - e^x \cancel{\sin x}$

$= \boxed{2e^x \cos x}$

112.



The volume of a cone is $\frac{1}{3}\pi r^2 h$.

The length of the arc cut from the original circle is 12θ, so the remaining circumference is $24\pi - 12\theta = 2\pi r$ = circumference of the circle at the base of the cone

So $r = \dfrac{24\pi - 12\theta}{2\pi} = 12 - \dfrac{6}{\pi}\theta$

Using the Pythagorean thm, $h^2 + r^2 = 144$, so

$h^2 + \cancel{144} - \dfrac{144\theta}{\pi} + \dfrac{36\theta^2}{\pi^2} = \cancel{144}$

and $h = \sqrt{\dfrac{144\theta}{\pi} - \dfrac{36\theta^2}{\pi^2}} = 6\sqrt{\dfrac{4\theta}{\pi} - \dfrac{\theta^2}{\pi^2}}$

112 cont.

So $V(\theta) = \frac{1}{3}\pi\left(12 - \frac{6}{\pi}\theta\right)^2 \cdot \cancel{6}^{2} \sqrt{\frac{4\theta}{\pi} - \frac{\theta^2}{\pi^2}}$

$= 2\pi\left(12 - \frac{6}{\pi}\theta\right)^2 \sqrt{\frac{4\theta}{\pi} - \frac{\theta^2}{\pi^2}}$

To find the angle θ which maximizes V, take the derivative and set it equal to 0.

$$0 = V'(\theta) = 2\cancel{\pi} \cdot 2\left(12 - \frac{6}{\pi}\theta\right)\left(-\frac{6}{\cancel{\pi}}\right)\sqrt{\frac{4\theta}{\pi} - \frac{\theta^2}{\pi^2}} + 2\pi\left(12 - \frac{6\theta}{\pi}\right)^2 \cdot \frac{1}{2}\left(\frac{4\theta}{\pi} - \frac{\theta^2}{\pi^2}\right)^{-1/2}\left(\frac{4}{\pi} - \frac{2\theta}{\pi^2}\right)$$

$$= -144\left(2 - \frac{\theta}{\pi}\right)\left(\frac{4\theta}{\pi} - \frac{\theta^2}{\pi^2}\right)^{1/2} + 72\left(2 - \frac{\theta}{\pi}\right)^3\left(\frac{4\theta}{\pi} - \frac{\theta^2}{\pi^2}\right)^{-1/2}$$

So $$0 = \left(2 - \frac{\theta}{\pi}\right)\left(-2\left(\frac{4\theta}{\pi} - \frac{\theta^2}{\pi^2}\right)^{1/2} + \left(2 - \frac{\theta}{\pi}\right)^2\left(\frac{4\theta}{\pi} - \frac{\theta^2}{\pi^2}\right)^{-1/2}\right)$$

If $V'(\theta) = 0$, $2 - \frac{\theta}{\pi} = 0$ or $-2\left(\frac{4\theta}{\pi} - \frac{\theta^2}{\pi^2}\right)^{1/2} + \left(2 - \frac{\theta}{\pi}\right)^2\left(\frac{4\theta}{\pi} - \frac{\theta^2}{\pi^2}\right)^{-1/2} = 0$.

If $2 - \frac{\theta}{\pi} = 0$, $\theta = 2\pi$. This must be a minimum. Geometrically, $\theta = 2\pi$ means that we remove the whole circle, so $V(2\pi) = 0$.

If $-2\left(\frac{4\theta}{\pi} - \frac{\theta^2}{\pi^2}\right)^{1/2} + \left(2 - \frac{\theta}{\pi}\right)^2\left(\frac{4\theta}{\pi} - \frac{\theta^2}{\pi^2}\right)^{-1/2} = 0$,

$$\left(2 - \frac{\theta}{\pi}\right)^2\left(\frac{4\theta}{\pi} - \frac{\theta^2}{\pi^2}\right)^{-1/2} = 2\left(\frac{4\theta}{\pi} - \frac{\theta^2}{\pi^2}\right)^{1/2}$$

$$\left(2 - \frac{\theta}{\pi}\right)^2 = 2\left(\frac{4\theta}{\pi} - \frac{\theta^2}{\pi^2}\right)$$

$$4 - \frac{4\theta}{\pi} + \frac{\theta^2}{\pi^2} = \frac{8\theta}{\pi} - \frac{2\theta^2}{\pi^2}$$

$$3\theta^2 - 12\pi\theta + 4\pi^2 = 0$$

Using the quadratic equation, $\theta = \frac{12\pi \pm \sqrt{144\pi^2 - 48\pi^2}}{6}$

$= 2\pi \pm \frac{\sqrt{96}\,\pi}{6}$

$= 2\pi \pm \frac{2\sqrt{6}}{3}\pi$

We can't take away more than 2π of the circle, so the maximum must be at

$$\boxed{\theta = 2\pi - \frac{2\sqrt{6}}{3}\pi}$$

Section 11.3

Odds

9. $\int \pi \sin \pi x \, dx$

Use substitution: $u = \pi x$

Then $du = \pi dx$

$$\int \pi \sin \pi x \, dx = \int \sin u \, du = -\cos u + C = \boxed{-\cos \pi x + C}$$

13. $\int \frac{1}{\theta^2} \cos \frac{1}{\theta} \, d\theta$

Use substitution: $u = \frac{1}{\theta}$

Then $du = -\frac{1}{\theta^2} d\theta \Rightarrow -du = \frac{1}{\theta^2} d\theta$

$$\int \frac{1}{\theta^2} \cos \frac{1}{\theta} \, d\theta = -\int \cos u \, du = -\sin u + C = \boxed{-\sin \frac{1}{\theta} + C}$$

15. $\int \sin 2x \cos 2x \, dx$

Use a trig identity: $\sin 2x \cos 2x = \frac{1}{2} \sin(2(2x)) = \frac{1}{2} \sin 4x$

$$\int \sin 2x \cos 2x \, dx = \frac{1}{2} \int \sin 4x \, dx \qquad (u = 4x, \ du = 4dx)$$

$$= \frac{1}{2} \cdot \frac{1}{4} \int \sin u \, du$$

$$= -\frac{1}{8} \cos u + C = \boxed{-\frac{1}{8} \cos 4x + C}$$

31. $\int \frac{\sec x \tan x}{\sec x - 1} \, dx$

Use substitution: $u = \sec x - 1$

$du = \sec x \tan x \, dx$

$$\int \frac{\sec x \tan x}{\sec x - 1} \, dx = \int \frac{1}{u} \, du = \ln|u| + C = \boxed{\ln|\sec x - 1| + C}$$

61. $y = 2\sin x + \sin 2x$

$$\int_0^\pi 2\sin x + \sin 2x \, dx = 2\int_0^\pi \sin x \, dx + \int_0^\pi \sin 2x \, dx$$

$$= (-2\cos x)\Big|_0^\pi + \frac{1}{2} \int_{x=0}^{x=\pi} \sin u \, du \qquad (u = 2x, \ du = 2dx)$$

$$= (-2\cos \pi - -2\cos 0) + -\frac{1}{2} \cos 2x \Big|_0^\pi$$

$$= -2(-1) + 2 + \left(-\frac{1}{2}\cos 2\pi - -\frac{1}{2}\cos 0\right)$$

$$= 2 + 2 + \left(-\cancel{\tfrac{1}{2}} + \cancel{\tfrac{1}{2}}\right)$$

$$= \boxed{4}$$

Evens

6. $\int \sec y(\tan y - \sec y)\, dy = \int \sec y \tan y - \sec^2 y \, dy$

$= \int \sec y \tan y - \int \sec^2 y \, dy = \boxed{\sec y - \tan y + C}$

10. $\int 4x^3 \sin x^4 \, dx$

Use substitution: $u = x^4$

$du = 4x^3 \, dx$

$\int 4x^3 \sin x^4 \, dx = \int \sin u \, du = -\cos u + C = \boxed{-\cos x^4 + C}$

18. $\int \sqrt{\tan x} \, \sec^2 x \, dx$

Use substitution: $u = \tan x$

$du = \sec^2 x \, dx$

$\int \sqrt{\tan x} \, \sec^2 x \, dx = \int \sqrt{u} \, du = \frac{2}{3} u^{3/2} + C = \boxed{\frac{2}{3} (\tan x)^{3/2} + C}$

20. $\int \frac{\sin x}{\cos^3 x} \, dx$

Use substitution: $u = \cos x$

$du = -\sin x \, dx$

$\int \frac{\sin x}{\cos^3 x} \, dx = -\int \frac{1}{u^3} \, du = \frac{-u^{-2}}{-2} + C = \boxed{\frac{1}{2} (\cos x)^{-2} + C}$

24. $\int e^{\sin x} \cos x \, dx$

Use substitution: $u = \sin x$

$du = \cos x \, dx$

$\int e^{\sin x} \cos x \, dx = \int e^u \, du = e^u + C = \boxed{e^{\sin x} + C}$

34. $\int_0^{\pi/4} \frac{1 - \sin^2\theta}{\cos^2\theta} \, d\theta = \int_0^{\pi/4} \frac{\cos^2\theta}{\cos^2\theta} \, d\theta = \int_0^{\pi/4} 1 \, d\theta = \theta \Big|_0^{\pi/4} = \boxed{\pi/4}$

38. $\int_{-\pi/2}^{\pi/2} (2t + \cos t)\, dt = \int_{-\pi/2}^{\pi/2} 2t \, dt + \int_{-\pi/2}^{\pi/2} \cos t \, dt$

$= t^2 \Big|_{-\pi/2}^{\pi/2} + \sin t \Big|_{-\pi/2}^{\pi/2}$

$= \frac{\pi^2}{4} - \left(\frac{-\pi}{2}\right)^2 + \sin\frac{\pi}{2} - \sin\left(-\frac{\pi}{2}\right)$

$= \frac{\pi^2}{4} - \frac{\pi^2}{4} + 1 - (-1) = 0 + 2 = \boxed{2}$

60. $y = x + \sin x$.

$$\int_0^\pi x + \sin x \, dx = \int_0^\pi x \, dx + \int_0^\pi \sin x \, dx = \frac{x^2}{2}\Big|_0^\pi + (-\cos x)\Big|_0^\pi$$

$$= \frac{\pi^2}{2} - 0 + -\cos\pi - -\cos 0$$

$$= \frac{\pi^2}{2} - (-1) + (1) = \boxed{\frac{\pi^2}{2} + 2}$$

62. $y = \sin x + \cos 2x$

$$\int_0^\pi \sin x + \cos 2x \, dx = \int_0^\pi \sin x \, dx + \int_0^\pi \cos 2x \, dx \qquad (u = 2x,\ du = 2dx)$$

$$= -\cos x\Big|_0^\pi + \frac{1}{2}\sin 2x\Big|_0^\pi$$

$$= -(-1) - -1 + \frac{1}{2}\sin 2\pi - \frac{1}{2}\sin 0.$$

$$= \boxed{2}$$

84. $R = 2.876 + 2.202\sin(0.576t + 0.847)$

a) Find extrema over a one year period (i.e. $t=0$ to $t=12$)

$$R' = 2.202\cos(0.576t + 0.847) \cdot 0.576$$

$$= 1.268352\cos(0.576t + 0.847) = 0$$

This is 0 when $\cos(0.576t + 0.847) = 0$.

$\cos x = 0$ if $x = \frac{\pi}{2} + 2\pi k$ or $\frac{3\pi}{2} + 2\pi k$ (for k an integer)

$$0.576t + 0.847 = \frac{\pi}{2} + 2\pi k \Rightarrow t \approx \boxed{1.26} + 10.9k$$

$$0.576t + 0.847 = \frac{3\pi}{2} + 2\pi k \Rightarrow t \approx \boxed{6.71} + 10.9k$$

Extrema: $R = 2.876 + 2.202\sin(0.576(1.26) + 0.847) = \boxed{5.08 \text{ in}}$ (in early Feb)

$R = 2.876 + 2.202\sin(0.576(6.71) + 0.847) = \boxed{0.674 \text{ in}}$ (in late July)

b) $$\int_0^{12} 2.876 + 2.202\sin(0.576t + 0.847) \, dt$$

$$= 2.876t\Big|_0^{12} + \frac{-2.202}{0.576}\cos(0.576t + 0.847)\Big|_0^{12}$$

$$= 34.512 - (-2.169) \approx \boxed{36.68 \text{ in}}$$

c) Avg monthly precipitation for Oct., Nov. and Dec.: $t = 9$ through $t = 12$

$$\frac{1}{3}\int_9^{12} 2.876 + 2.202\sin(0.576t + 0.847)\,dt = \frac{1}{3}\left[2.876t\Big|_9^{12} + \frac{2.202}{-0.576}\cos(0.576t + 0.847)\Big|_9^{12}\right]$$

$$\approx \frac{1}{3}(11.967) \approx \boxed{3.99 \text{ in}}$$