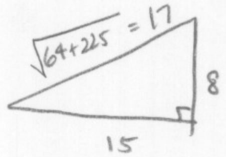
Homework 10 Solutions

Section 10.5 odds

43. $\cos^4 x = (\cos^2 x)^2 = \left(\frac{1+\cos 2x}{2}\right)^2 = \frac{1}{4}\left(1 + 2\cos 2x + \cos^2 2x\right)$

$= \frac{1}{4}\left(1 + 2\cos 2x + \frac{1+\cos 4x}{2}\right) = \boxed{\frac{1}{8}\left(3 + 4\cos 2x + \cos 4x\right)}$

53. $\cos\left(\frac{\theta}{2}\right) = \sqrt{\frac{1+\cos\theta}{2}} = \sqrt{\frac{1+\frac{15}{17}}{2}} = \sqrt{\frac{16}{17}} = \boxed{\frac{4\sqrt{17}}{17}}$



63. $\sin\frac{\pi}{8} = \sin\frac{\pi/4}{2} = \sqrt{\frac{1-\cos\frac{\pi}{4}}{2}} = \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}} = \boxed{\frac{\sqrt{2-\sqrt{2}}}{2}}$

$\cos\frac{\pi}{8} = \cos\frac{\pi/4}{2} = \sqrt{\frac{1+\cos\frac{\pi}{4}}{2}} = \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = \boxed{\frac{\sqrt{2+\sqrt{2}}}{2}}$

67. $\cos u = \frac{7}{25}$, $0 < u < \frac{\pi}{2}$

a) Since $0 < u < \frac{\pi}{2}$, $0 < \frac{u}{2} < \frac{\pi}{4}$, so $\frac{u}{2}$ lies in quadrant I.

b) $\sin\frac{u}{2} = \sqrt{\frac{1-\cos u}{2}} = \sqrt{\frac{1-\frac{7}{25}}{2}} = \sqrt{\frac{9}{25}} = \boxed{\frac{3}{5}}$

$\cos\frac{u}{2} = \sqrt{\frac{1+\cos u}{2}} = \sqrt{\frac{1+\frac{7}{25}}{2}} = \sqrt{\frac{16}{25}} = \boxed{\frac{4}{5}}$

Evens

48. $\sin^2 x \cos^4 x = \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1}{4}\left(3+4\cos 2x + \cos 4x\right)\right)$ (from #43)

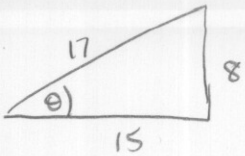
$= \frac{1}{8}\left(3 + \cos 2x - 4\cos^2 2x + \cos 4x - (\cos 2x)(\cos 4x)\right)$

$= \frac{1}{8}\left(3 + \cos 2x - \cancel{4}^{2}\left(\frac{1+\cos 4x}{\cancel{2}}\right) + \cos 4x - (\cos 2x)(\cos 4x)\right)$

$= \frac{1}{8}\left(1 + \cos 2x - \cos 4x - (\cos 2x)(\cos 4x)\right)$

$= \frac{1}{8}\left(1 + \cos 2x - \cos 4x - \frac{1}{2}(\cos 2x + \cos 6x)\right)$ (Using product-to-sum formulas)

$= \boxed{\frac{1}{16}\left(2 + \cos 2x - 2\cos 4x - \cos 6x\right)}$

54. 

$$\sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}} = \sqrt{\frac{1-\frac{8}{17}}{2}} = \sqrt{\frac{9}{34}} = \boxed{\frac{3\sqrt{34}}{34}}$$

60. $$\sin 165° = \sin\frac{330°}{2} = +\sqrt{\frac{1-\cos 330°}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \boxed{\frac{\sqrt{2-\sqrt{3}}}{2}}$$

$$\cos 165° = \cos\frac{330°}{2} = -\sqrt{\frac{1+\cos 330°}{2}} = -\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = \boxed{-\frac{\sqrt{2+\sqrt{3}}}{2}}$$

64. $$\sin\frac{\pi}{12} = \sin\frac{\pi/6}{2} = \sqrt{\frac{1-\cos\frac{\pi}{6}}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \boxed{\frac{\sqrt{2-\sqrt{3}}}{2}}$$

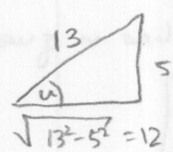
$$\cos\frac{\pi}{12} = \cos\frac{\pi/6}{2} = \sqrt{\frac{1+\cos\frac{\pi}{6}}{2}} = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = \boxed{\frac{\sqrt{2+\sqrt{3}}}{2}}$$

68. $\sin u = \frac{5}{13}$, $\frac{\pi}{2} < u < \pi$.

a) $\frac{\pi}{2} < u < \pi$. so $\frac{\pi}{4} < \frac{u}{2} < \frac{\pi}{2}$ and $\frac{u}{2}$ is in the first quadrant.

b) $\sin\frac{u}{2} = \sqrt{\frac{1-\cos u}{2}}$

Since $\sin u = \frac{5}{13}$,



$\cos u = -\frac{12}{13}$ (since u in quadrant II, cos is negative).

Then $$\sin\frac{u}{2} = \sqrt{\frac{1+\frac{12}{13}}{2}} = \sqrt{\frac{25}{26}} = \boxed{\frac{5\sqrt{26}}{26}}$$

$$\cos\frac{u}{2} = \sqrt{\frac{1+\cos u}{2}} = \sqrt{\frac{1+\frac{-12}{13}}{2}} = \sqrt{\frac{1}{26}} = \boxed{\frac{\sqrt{26}}{26}}$$

72. $\sec u = \frac{7}{2}$, $\frac{3\pi}{2} < u < 2\pi$.

a) Since $\frac{3\pi}{2} < u < 2\pi$, $\frac{3\pi}{4} < \frac{u}{2} < \pi$ and $\frac{u}{2}$ is in quadrant II.

b) $$\sin\frac{u}{2} = \sqrt{\frac{1-\cos u}{2}} = \sqrt{\frac{1-\frac{1}{\sec u}}{2}} = \sqrt{\frac{1-\frac{2}{7}}{2}} = \boxed{\sqrt{\frac{5}{14}}}$$

$$\cos\frac{u}{2} = -\sqrt{\frac{1+\cos u}{2}} = -\sqrt{\frac{1+\frac{2}{7}}{2}} = -\sqrt{\frac{9}{14}} = \boxed{\frac{-3\sqrt{14}}{14}}$$

Section 11.1

Odds

11. $\lim_{x\to 0} \sec 2x = \sec(2\cdot 0) = \sec 0 = \frac{1}{\cos 0} = \frac{1}{1} = \boxed{1}$

41. $\lim_{x\to 0} \frac{\sin x(1-\cos x)}{x^2} = \left(\lim_{x\to 0} \frac{\sin x}{x}\right)\left(\lim_{x\to 0} \frac{1-\cos x}{x}\right)$

$= 1\cdot 0 = \boxed{0}$

Evens

12. $\lim_{x\to \pi} \cos 3x = \cos 3\pi = \boxed{-1}$

40. $\lim_{x\to 0} \frac{3(1-\cos x)}{x} = 3\cdot\left(\lim_{x\to 0}\frac{1-\cos x}{x}\right)$

$= 3\cdot 0 = \boxed{0}$

Section 11.2

Odds.

9. $f(x) = x^3 \cos x$

$f'(x) = 3x^2\cos x + x^3(-\sin x)$ (Product rule)

$= \boxed{3x^2\cos x - x^3\sin x}$

15. $g(x) = \frac{\sin x}{x^2}$

$g'(x) = \frac{x^2\cos x - 2x\sin x}{x^4}$ (quotient rule)

$= \boxed{\frac{x\cos x - 2\sin x}{x^3}}$

17. $g(\theta) = \frac{\theta}{1-\sin\theta}$

$g'(\theta) = \frac{(1-\sin\theta)\cdot 1 - \theta(0-\cos\theta)}{(1-\sin\theta)^2}$ (quotient rule)

$= \boxed{\frac{1-\sin\theta + \theta\cdot\cos\theta}{(1-\sin\theta)^2}}$

27. $f(x) = \sin x \cos x$, $(\frac{\pi}{2}, 0)$.

$f'(x) = (\sin x)(-\sin x) + (\cos x)(\cos x)$ (product rule)

$= \cos^2 x - \sin^2 x$

$= \cos 2x$.

$f'(\frac{\pi}{2}) = \cos \pi = -1$

So the equation of the line through $(\frac{\pi}{2}, 0)$ tangent to $f(x) = \sin x \cos x$

is $y = -(x - \frac{\pi}{2})$, i.e. $\boxed{y = -x + \frac{\pi}{2}}$

Evens

10. $g(x) = \sqrt{x} \sin x$

$g'(x) = \frac{1}{2} x^{-1/2} \sin x + \sqrt{x} \cos x$ (product rule)

$= \boxed{\frac{\sin x}{2\sqrt{x}} + \sqrt{x} \cos x}$

14. $f(x) = \frac{\sin x}{x}$.

$f'(x) = \frac{x \cos x - 1 \sin x}{x^2}$ (quotient rule)

$= \boxed{\frac{x \cos x - \sin x}{x^2}}$

16. $f(t) = \frac{\cos t}{t^3}$

$f'(t) = \frac{t^3(-\sin t) - 3t^2 \cos t}{t^6}$ (quotient rule)

$= \boxed{\frac{-t \sin t - 3 \cos t}{t^4}}$

18. $f(\theta) = \frac{\sin \theta}{1 - \cos \theta}$.

$f'(\theta) = \frac{(1 - \cos \theta) \cos \theta - (\sin \theta)(\sin \theta)}{(1 - \cos \theta)^2}$ (quotient rule).

$= \frac{\cos \theta - \cos^2 \theta - \sin^2 \theta}{(1 - \cos \theta)^2} = \frac{\cos \theta - (\sin^2 \theta + \cos^2 \theta)}{(1 - \cos \theta)^2} = \frac{\cos \theta - 1}{(1 - \cos \theta)^2}$

$= \boxed{\frac{-1}{1 - \cos \theta}}$

28. $f(x) = x\sin x + \cos x$, $(\pi, -1)$.

$f'(x) = \cancel{\sin x} + x\cos x - \cancel{\sin x} = x\cos x$.

$f'(\pi) = \pi\cos\pi = \pi(-1) = -\pi$.

So the line through $(\pi, -1)$ tangent to the graph of $f(x)$ is

$y + 1 = -\pi(x - \pi)$

or in other words

$\boxed{y = -\pi x + \pi^2 - 1}$

30. $f(x) = \sin(\cos x)$, $(\frac{3\pi}{2}, 0)$

$f'(x) = [\cos(\cos x)] \cdot (-\sin x)$ (Chain rule).

$f'(\frac{3\pi}{2}) = [\cos(\cos\frac{3\pi}{2})] \cdot (-\sin\frac{3\pi}{2})$

$= \cos(0) \cdot (+1)$

$= 1$

So the line through $(\frac{3\pi}{2}, 0)$ tangent to the graph of $f(x)$ is

$\boxed{y = x - \frac{3\pi}{2}}$

32. $y = \sin\pi x$

$y' = (\cos\pi x)\pi$ (chain rule

$= \boxed{\pi\cos\pi x}$

36. $y = \cos(1-2x)^2$

$y' = \left(-\sin(1-2x)^2\right)\left(2(1-2x)\right)(-2)$ Chain rule.

$= \boxed{4(1-2x)\sin(1-2x)^2}$

44. $g(t) = 5\cos^2\pi t = 5(\cos\pi t)^2$

$g'(t) = 5 \cdot (2\cos\pi t)(-\sin\pi t) \cdot \pi$

$= \boxed{-10\pi\sin\pi t\cos\pi t} = \boxed{-5\pi\sin 2\pi t}$

68. $y = e^x(3\cos 2x - 4\sin 2x)$.

Show y satisfies $y'' - 2y' + 5y = 0$.

$$y' = e^x(3\cos 2x - 4\sin 2x) + e^x((-3\sin 2x)2 - (4\cos 2x)\cdot 2)$$
$$= e^x(-5\cos 2x - 10\sin 2x).$$

$$y'' = e^x(-5\cos 2x - 10\sin 2x) + e^x((5\sin 2x)\cdot 2 - (10\cos 2x)\cdot 2)$$
$$= e^x(-25\cos 2x)$$

$$y'' - 2y' + 5y = e^x(-25\cos 2x) - 2e^x(-5\cos 2x - 10\sin 2x)$$
$$+ 5e^x(3\cos 2x - 4\sin 2x)$$
$$= e^x((-25+10+15)\cos 2x + (20-20)\sin 2x)$$
$$= e^x(0+0) = 0$$

So $y'' - 2y' + 5y = 0$, as desired