MATH 100 – PRACTICE EXAM 3

Name:____

Lecture:	MWF 12	MWF 1	MWF 3
Recitation:	T. Crawford Th 11	T. Crawford Th 1	T. Crawford Th 2
	G. Chiloyan Th 3	G. Chiloyan Th 11	G. Chiloyan Th 12

FOR FULL CREDIT, SHOW ALL WORK
NO CALCULATORS

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2	
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1. Let $f(x) = \arctan x$.

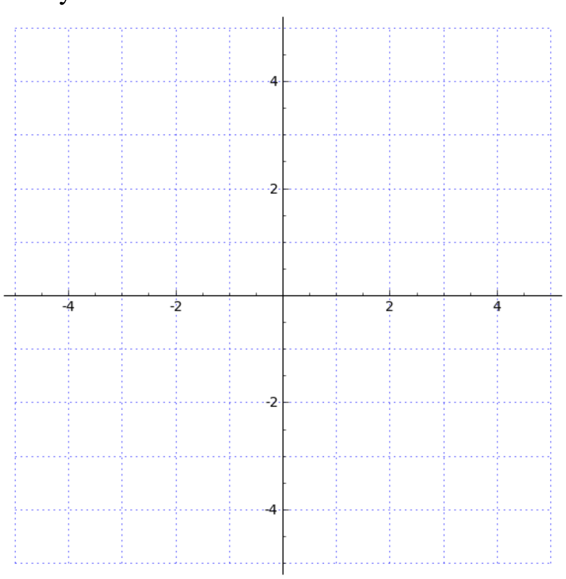
(a) Find the location of all local minima and local maxima for f. (3 points)

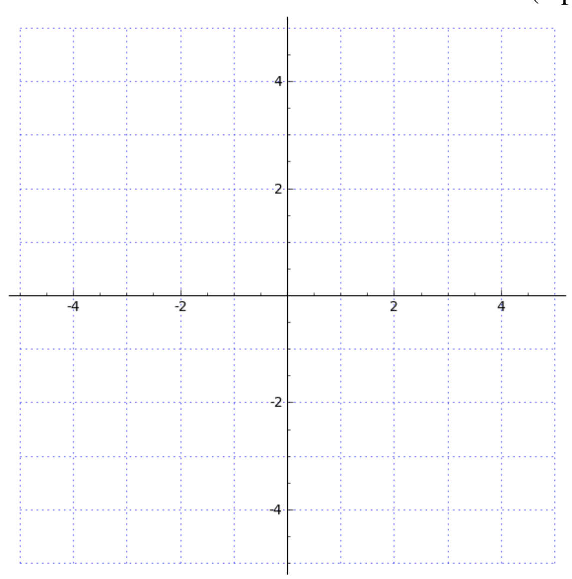
(b) Find the intervals where f is increasing and where f is decreasing. (2 points)

(c) Find the intervals where f is concave up and the intervals where f is concave down. (3 points)

(d) Find all points of inflection for f. (2 points)

(e) Sketch a graph of $f(x)$ and of $f'(x)$. Indicate which is the graph of f and which is the graph of f' clearly. (5 points)





2. Find all points (x, y) where the line tangent to the curve defined by

$$x^2 - \frac{x}{y^2+1} = \frac{1}{3}$$

are horizontal. (8 points)

3. Compute $g'(1)$ where $g(x)$ is the inverse function of $f(x) = x^{\log_2 x}$. (Hint: Express f as a power of e or use logarithmic differentiation.) (9 points)

4. Use Rolle's Theorem to explain why $f(x) = x^5 + 16x$ has exactly one real root. (7 points)

5. A cylindrical roll of toilet paper is 6 inches wide. Suppose you use the toilet paper at a constant rate (that is, the volume of the toilet paper roll changes at a constant rate of $-k$ cubic inches per week).

   (a) How quickly is the radius of the toilet paper roll changing when the radius is 2 inches? Include units and state your answer in a complete sentence. (7 points)

   (b) Does the radius change more quickly at the beginning of the life of the toilet paper roll or near the end (that is, when the radius is larger or smaller)? (3 points)

6. Production of toys at Santa's workshop follows the Cobb-Douglas production formula

$$P = 2L^{3/2}K$$

where P is the production level, L is the labor cost (in billions of dollars), and K is the cost of equipment (a.k.a capital, in billions of dollars). Suppose that this year, Santa must produce 2 billion toys for the good children of the world.

(a) At what rate can Santa replace labor with capital? That is, if labor changes a little, how must capital change in response to keep production levels at the target? (2 points)

(b) Suppose that labor cost is capped at 1 billion dollars (because of a shortage of skilled elves). How much should Santa spend on labor and on equipment to minimize his total cost, $L+K$? To solve this problem, are you optimizing over an open or closed interval? (9 points)

Formula Sheet

You may use these formulas when appropriate.

$\sin(\pi/2 - x) = \cos x$

$\cos(\pi/2 - x) = \sin x$

$\sin(-x) = -\sin x$

$\cos(-x) = \cos x$

$\sin(x + y) = \sin x \cos y + \cos x \sin y$

$\sin(x - y) = \sin x \cos y - \cos x \sin y$

$\cos(x + y) = \cos x \cos y - \sin x \sin y$

$\cos(x - y) = \cos x \cos y + \sin x \sin y$

$\sin 2x = 2 \sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

Volume of a sphere $= \frac{4}{3}\pi r^3$

Surface area of a sphere $= 4\pi r^2$

Volume of a cylinder $= \pi r^2 h$

Surface area of a cylinder $= 2\pi r^2 + 2\pi r h$

Volume of a cone $= \frac{1}{3}\pi r^2 h$