MATH 100 – PRACTICE EXAM 1

Name: SOLUTIONS.

Lecture:	MWF 12	MWF 1	MWF 3
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FOR FULL CREDIT, SHOW ALL WORK
NO CALCULATORS

1	
2	
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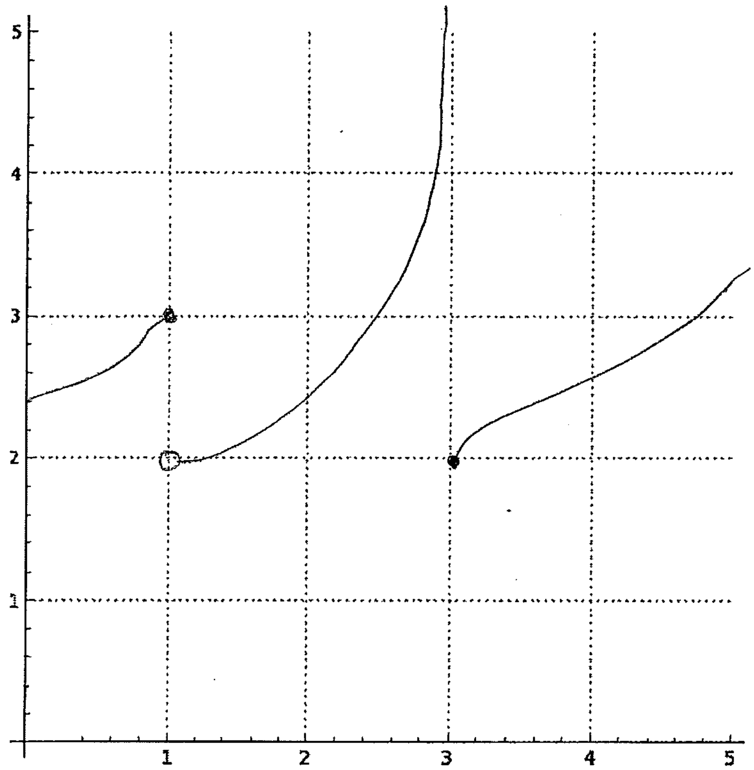
1. (a) Sketch the graph of a function with the following properties. (8 points)

   (i) $\lim_{x\to 1} f(x)$ does not exist

   (ii) $f(1)$ is defined

   (iii) $\lim_{x\to 3^+} f(x) = 2$

   (iv) $\lim_{x\to 3^-} f(x) = \infty$



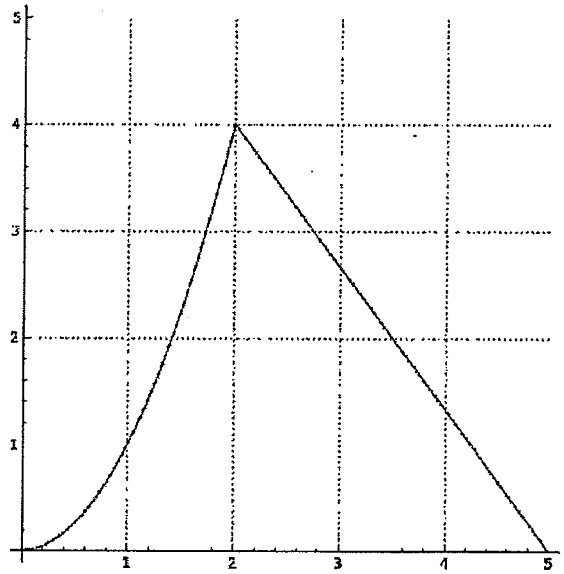
(b) What are the possible values of $f(3)$? (2 points)

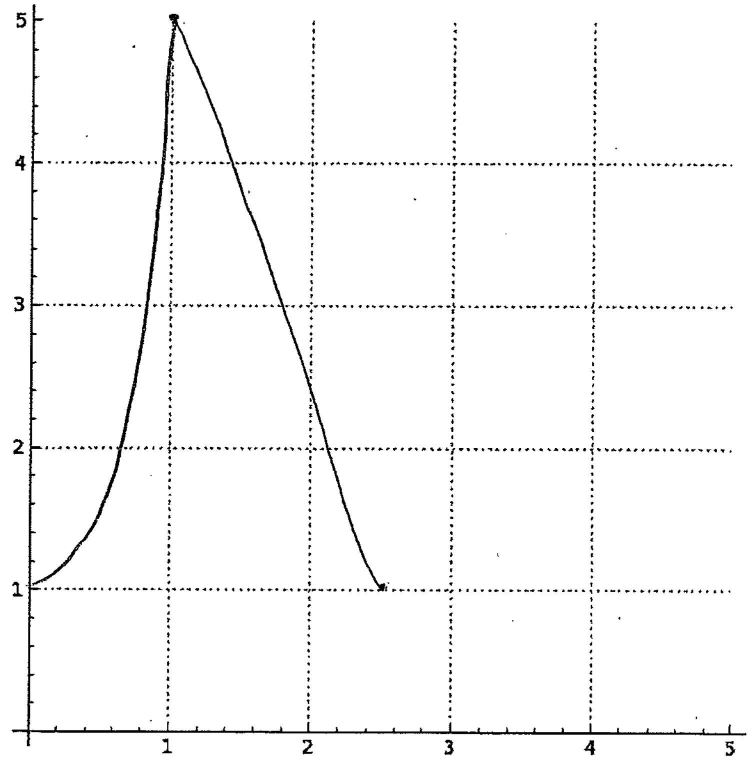
$f(3)$ can be any real number.

($f(3)$ need not be equal to $\lim_{x\to 3^+} f(x)$ or $\lim_{x\to 3^-} f(x)$.

In fact, $f(3)$ need not even be defined).

2. Sketch the graph of $f(2x)+1$, given the following graph for $f(x)$. (10 points)





(Compressed horizontally by a factor of 2 and shifted up by 1)

3. Solve for x. (10 points)

$$3^{2\log_3 x} - 2\ln e^x = 3$$

$3^{2\log_3 x} = \left(3^{\log_3 x}\right)^2 = x^2$

$-2\ln e^x = -2x$

So $x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

So $x = 3$ or $x = -1$.

BUT! $x \neq -1$, since $\log_3(-1)$ is not defined.

So only $\boxed{x=3}$ makes sence

4. Using the basic limit laws, compute the following limit. *You must show at least five of the steps you are using to receive full credit.* (10 points)

$$\lim_{t\to 2}(5t^2 + (t+1)\sqrt{t-1})$$

$= \lim_{t\to 2} 5t^2 + \lim_{t\to 2}(t+1)\sqrt{t-1} = 5\lim_{t\to 2} t^2 + \left(\lim_{t\to 2}(t+1)\right)\left(\lim_{t\to 2}\sqrt{t-1}\right)$

$= 5\left(\lim_{t\to 2} t\right)^2 + \left(\lim_{t\to 2} t + \lim_{t\to 2} 1\right)\left(\sqrt{\lim_{t\to 2}(t-1)}\right)$

$= 5\cdot 2^2 + (2+1)\sqrt{\lim_{t\to 2} t - \lim_{t\to 2} 1}$

$= 20 + 3\sqrt{2-1}$

$= 20 + 3\sqrt{1} = \boxed{23}$

5. (a) Find the inverse function of

$$f(x) = \frac{1}{\sqrt{x+1}}.$$

(6 points)

$$y = \frac{1}{\sqrt{x+1}}$$

$$\sqrt{x+1} = \frac{1}{y}$$

So $x+1 = \frac{1}{y^2}$ and $x = \frac{1}{y^2} - 1$.

Therefore $f^{-1}(y) = \frac{1}{y^2} - 1$.

(b) Verify that you got the correct inverse function in part (a) by composing your answer $g(y)$ with $f(x)$. That is, compute $f \circ g$ and $g \circ f$. (4 points)

$$f \circ g(y) = f\left(\frac{1}{y^2} - 1\right) = \frac{1}{\sqrt{\frac{1}{y^2} - 1 + 1}} = \frac{1}{\sqrt{\frac{1}{y^2}}} = \frac{1}{\left(\frac{1}{y}\right)} = y \checkmark$$

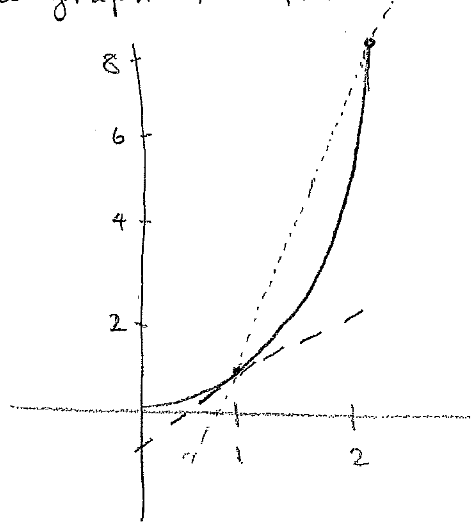
$$g \circ f(x) = g\left(\frac{1}{\sqrt{x+1}}\right) = \frac{1}{\left(\frac{1}{\sqrt{x+1}}\right)^2} - 1$$

$$= \frac{1}{\frac{1}{x+1}} - 1 = x + 1 - 1 = x \checkmark$$

6. Find the average rate of change of the function $f(x) = x^3$ on the range $[1, 2]$. Is this average rate of change greater than or less than the instantaneous rate of change at $x = 1$? (10 points)

Average rate of change: $\frac{\Delta y}{\Delta x} = \frac{2^3 - 1^3}{2-1} = \frac{7}{1} = 7.$

The graph of $f(x) = x^3$ looks like:



The average rate of change is the slope of the dotted secant line.

The instantaneous rate of change is the slope of the dashed line tangent to the graph of f at $x = 1$.

The slope of the secant line is greater, so the average rate of change is greater than the instantaneous rate of change at $x = 1$.