Homework 9 (Due 11/8)

Solutions

Section 3.9

38. $y = (3x+5)(4x+9)$

$\ln y = \ln(3x+5) + \ln(4x+9)$

$\frac{d}{dx}(\ln y) = \frac{y'}{y} = \frac{3}{3x+5} + \frac{4}{4x+9}$

So $y' = (3x+5)(4x+9)\left[\frac{3}{3x+5} + \frac{4}{4x+9}\right] = \boxed{3(4x+9) + 4(3x+5)}$

40. $y = \frac{x(x+1)^3}{(3x-1)^2}$

$\ln y = \ln x + 3\ln(x+1) - 2\ln(3x-1)$

$\frac{d}{dx}(\ln y) = \frac{y'}{y} = \frac{1}{x} + \frac{3}{x+1} - 2\frac{3}{3x-1}$

So $y' = \boxed{\frac{x(x+1)^3}{(3x-1)^2}\left(\frac{1}{x} + \frac{3}{x+1} - \frac{6}{3x-1}\right)}$

42. $y = (2x+1)(4x^2)\sqrt{x-9}$

$\ln y = \ln(2x+1) + \ln(4x^2) + \frac{1}{2}\ln(x-9)$

$= \ln(2x+1) + \ln 4 + 2\ln x + \frac{1}{2}\ln(x-9)$

$\frac{d}{dx}(\ln y) = \frac{y'}{y} = \frac{2}{2x+1} + 0 + \frac{2}{x} + \frac{1}{2}\cdot\frac{1}{x-9}$

So $y' = (2x+1)(4x^2)\sqrt{x-9}\left(\frac{2}{2x+1} + \frac{2}{x} + \frac{1}{2(x-9)}\right)$

$= \boxed{8x^2\sqrt{x-9} + (2x+1)(8x)\sqrt{x-9} + \frac{(2x+1)(2x^2)}{\sqrt{x-9}}}$

43. $y = \sqrt{\frac{x(x+2)}{(2x+1)(3x+2)}}$

$\ln y = \frac{1}{2}\ln\left(\frac{x(x+2)}{(2x+1)(3x+2)}\right) = \frac{1}{2}\left(\ln x + \ln(x+2) - \ln(2x+1) - \ln(3x+2)\right)$

$\frac{d}{dx}\ln y = \frac{y'}{y} = \frac{1}{2}\left(\frac{1}{x} + \frac{1}{x+2} - \frac{2}{2x+1} - \frac{3}{3x+2}\right)$

So $y' = \boxed{\frac{1}{2}\sqrt{\frac{x(x+2)}{(2x+1)(3x+2)}}\left(\frac{1}{x} + \frac{1}{x+2} - \frac{2}{2x+1} - \frac{3}{3x+2}\right)}$

Section 3.10

Preliminary exercises

3. $x^2 + 2xy + y^3 = 7$.

Jason's answer for the derivative of the equation:

$$2x + 2xy' + 3y^2 = 0.$$

Jason has made two mistakes.

1) When he differentiates $2xy$, he needs to use the product rule since both x and y are functions of x. He should have gotten $2y + 2xy'$, not just $2xy'$.

2) He forgot the "y'" when differentiating $3y^2$. Since we're supposed to differentiate with respect to x, and y is a function of x, we need to use the chain rule and multiply by the derivative of y. He should have gotten $3y^2y'$, not just $3y^2$.

31. $xy + x^2y^2 = 5$ at $(2,1)$.

Differentiate:

$$xy' + y + 2xy^2 + 2x^2y\,y' = 0.$$

Plug in $x=2$, $y=1$:

$$2y' + 1 + 4 + 8y' = 0$$

$$10y' = -5$$

$$y' = -\frac{1}{2}.$$

So the tangent line has equation: $\boxed{y - 1 = -\frac{1}{2}(x-2)}$

36. $x^2e^y + ye^x = 4$ at $(2,0)$

Differentiate:

$$2xe^y + x^2e^y y' + y'e^x + ye^x = 0$$

Plug in $(2,0)$:

$$4e^0 + 4e^0 y' + y'e^2 + 0e^2 = 0$$

$$4 + 4y' + e^2y' = 0$$

$$y' = \frac{4}{4+e^2}.$$

So the tangent line has equation $y = \frac{4}{4+e^2}(x-2)$

37. $e^{2x-y} = \frac{x^2}{y}$ at $(2,4)$.

Differentiate:

$$e^{2x-y} \cdot (2 - y') = \frac{2yx - x^2 y'}{y^2}$$

Plug in $x=2$, $y=4$:

$$e^0 (2 - y') = \frac{16 - 4y'}{16}$$

$$2 - y' = 1 - \frac{1}{4} y'$$

$$1 = \frac{3}{4} y' \Rightarrow y' = \frac{4}{3}.$$

So the tangent line has equation $\boxed{y - 4 = \frac{4}{3}(x-2)}$

39. $y^2 = x^3 - 3x + 1$

a) Differentiate: $2yy' = 3x^2 - 3$.

b) When $y' = 0$: $0 = 3x^2 - 3$

So $x^2 = 1 \Rightarrow x = \pm 1$.

c) When $x=1$, $y^2 = 1 - 3 + 1 = -1$, which doesn't have a real solution, so $x=1$ doesn't correspond to a point on the graph.

d) When $x=-1$, $y^2 = -1 + 3 + 1 = 3$

so $y = \pm\sqrt{3}$.

The tangent lines are horizontal at the points $(-1, \sqrt{3})$ and $(-1, -\sqrt{3})$.

44. $x^3 + y^3 = 3xy$ at $(\frac{2}{3}, \frac{4}{3})$

Differentiate: $3x^2 + 3y^2 y' = 3y + 3xy'$

Plug in $x = \frac{2}{3}$, $y = \frac{4}{3}$: $3 \cdot (\frac{4}{9}) + 3(\frac{16}{9}) y' = 4 + 2y'$

$$\frac{4}{3} - 4 = (2 - \frac{16}{3}) y' \rightarrow -\frac{8}{3} = -\frac{10}{3} y'$$

So $y' = \frac{4}{5}$.

So the tangent line has equation $\boxed{y - \frac{4}{3} = \frac{4}{5}(x - \frac{2}{3})}$

45. $x^3 + y^3 = 3xy$: a point where the tangent line is horizontal?

$$3x^2 + 3y y' = 3y + 3xy'$$

Set $y' = 0$: $3x = 3y \Rightarrow x = y$.

Plug into the original equation: $x^3 + x^3 = 3x^2 \Rightarrow 2x^3 = 3x^2 \Rightarrow 2x = 3$, so $x = \frac{3}{2}$.

Since $x = y$, the point is $\boxed{(\frac{3}{2}, \frac{3}{2})}$