Homework 7: (Due 10/25)
Solutions

Section 3.4

8. $V = 20\sqrt{T}$

$$\frac{dV}{dT} = \frac{d}{dT}\left(20\,T^{1/2}\right) = \boxed{10\,T^{-1/2}}$$

14. $T(t) = \frac{3}{8}t^2 - 15t + 180$ for $0 \le t \le 20$

$T'(t) = \frac{3}{4}t - 15$

At $t = 10$, $T'(10) = \boxed{-\frac{15}{2} \text{ degrees Celsius per minute}}$

26. If the object is dropped, $v_0 = 0$ m/s

We can say that $s_0 = 300$ m and ground level is at 0.

Then $s(t) = s_0 + v_0 t - \frac{1}{2}gt^2$
$= 300 - \frac{1}{2}gt^2$

and $s'(t) = -gt$.

We need to know 1) when the object hits the ground
and 2) what its speed is at that time.

For 1), solve $s(t) = 0$ for t.

$$300 - \frac{1}{2}gt^2 = 0$$
$$t = \sqrt{\frac{600}{g}} \approx \sqrt{\frac{600}{9.8}} \approx 7.82 \text{ s}$$

For 2): plug this time in to $v(t) = s'(t)$

$v(t) = s'(t) = -gt$

So at 7.82 s, velocity is $(-9.8)(7.82) \approx \boxed{-76.68 \text{ m/s}}$

36. a) $A'(\text{Jan } 15) \approx \dfrac{A(\text{Feb}1) - A(\text{Jan } 1)}{31 \text{ days}}$ since there are 31 days in Jan.

$= \dfrac{1.23}{31} \approx 0.04$ ppm/day

$A'(\text{Feb } 15) \approx \dfrac{A(\text{March } 1) - A(\text{Feb } 1)}{28} = \dfrac{0.55}{28} \approx 0.02$ ppm/day

$A'(\text{March } 15) \approx \dfrac{A(\text{April } 1) - A(\text{March } 1)}{31} = \dfrac{2.03}{31} \approx 0.07$ ppm/day

$A'(\text{April } 15) \approx \dfrac{A(\text{May } 1) - A(\text{April } 1)}{30} = \dfrac{0.13}{30} \approx 0.004$ ppm/day

$A'(\text{May } 15) \approx \dfrac{A(\text{June } 1) - A(\text{May } 1)}{31} = \dfrac{-0.52}{31} \approx -0.02$ ppm/day

$A'(\text{June } 15) \approx \dfrac{A(\text{July } 1) - A(\text{June } 1)}{30} = \dfrac{-1.48}{30} \approx -0.05$ ppm/day

$A'(\text{July } 15) \approx \dfrac{A(\text{August } 1) - A(\text{July } 1)}{31} = \dfrac{-2.61}{31} \approx -0.08$ ppm/day

$A'(\text{Aug } 15) \approx \dfrac{A(\text{Sept } 1) - A(\text{Aug. } 1)}{31} = \dfrac{-1.05}{31} \approx -0.03$ ppm/day

$A'(\text{Sept } 15) \approx \dfrac{A(\text{Oct } 1) - A(\text{Sept } 1)}{30} = \dfrac{0.08}{30} \approx 0.003$ ppm/day

$A'(\text{Oct } 15) \approx \dfrac{A(\text{Nov } 1) - A(\text{Oct } 1)}{31} = \dfrac{1.52}{31} \approx 0.05$ ppm/day

$A'(\text{Nov } 15) \approx \dfrac{A(\text{Dec } 1) - A(\text{Nov } 1)}{30} = \dfrac{1.36}{30} \approx 0.05$ ppm/day

b) $A'(t)$ was largest in March
$A'(t)$ was smallest in July.

c) The CO_2 level was almost constant in September and April.

43. $C(x) = 300 + 0.25x - 0.5\left(\frac{x}{1000}\right)^3$.

Cost of producing 2000 bagels:

$$C(2000) = 300 + 0.25(2000) - 0.5\left(\frac{2000}{1000}\right)^3$$
$$= 300 + 500 - 0.5(2^3)$$
$$= 800 - 4 = \boxed{\$796}$$

To estimate the cost of the 2001ST bagel, use

marginal cost $\approx C'(x)$

$$C'(x) = 0.25 - \frac{0.5}{1000^3} \cdot 3x^2 = 0.25 - \frac{1.5}{1000^3}x^2.$$

$$C'(2000) = 0.25 - \frac{1.5}{1000^3} \cdot 2000^2$$
$$= 0.25 - \frac{1.5}{1000} \cdot 2^2 = 0.25 - \frac{6}{1000} = \boxed{\$0.244 \text{ per bagel}}$$

Actual cost of the 2001ST bagel:

$$C(2001) - C(2000) = 300 + 0.25(2001) - \left(\frac{2001}{1000}\right)^3 \cdot 0.5 - 796$$
$$\approx 796.243997 - 796$$
$$\approx \boxed{\$0.243997 \text{ per bagel}}$$

46. $f(p) = (34 - 0.612p)\,p^{-0.658}$

a) $f(15) = (34 - 0.612 \cdot 15)\,15^{-0.658} \approx \boxed{4.18 \text{ offspring per day}}$

$$f'(p) = -0.612\,p^{-0.658} + (34 - 0.612p)(-0.658)\,p^{-1.658}$$

So $f'(15) = -0.612\,(15^{-0.658}) + (34 - 0.612 \cdot 15)(-0.658)\,p^{-1.658}$

$\approx \boxed{-0.29}$ offspring/day per fly.

b) To estimate the decrease in offspring per female when the population is increased to 16 from 15, use $f'(15)$.

Each female produces about $\boxed{0.29}$ fewer offspring per day.

$f(16) - f(15) = (34 - 0.612 \cdot 16) \cdot 16^{-0.658} - 4.18 \approx \boxed{-0.27}$ offspring/day per fly

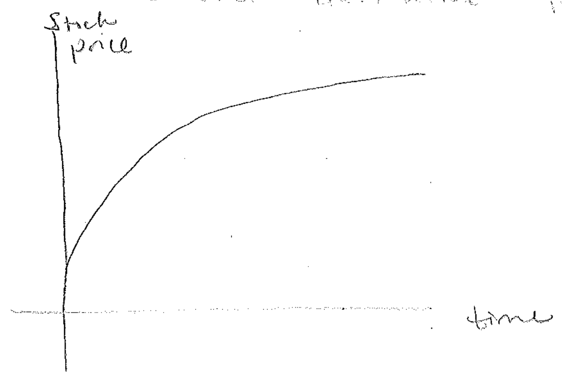
$-0.29 < -0.27$. The actual change is smaller in magnitude than the estimate.

Section 3.5

Preliminary Questions

1. "Stocks Go Higher, though the Pace of their gains slows"

This means the first derivative of the stock price function is positive, but the second derivative is negative.



Exercises

25. $h''(1)$, $h(w) = \sqrt{w}\, e^w$

$h(w) = w^{1/2} e^w$

$h'(w) = w^{1/2} e^w + \frac{1}{2} w^{-1/2} e^w$

$h''(w) = w^{1/2} e^w + \frac{1}{2} w^{-1/2} e^w + \frac{1}{2} w^{-1/2} e^w + -\frac{1}{4} w^{-3/2} e^w$

$h''(1) = e + e + e + e = \boxed{4e}$

26. $g''(0)$, $g(s) = \dfrac{e^s}{s+1}$

$$g'(s) = \frac{(s+1)e^s - e^s}{(s+1)^2} = \frac{e^s(s \cancel{+1} \cancel{-1})}{(s+1)^2} = \frac{se^s}{(s+1)^2}$$

$$g''(s) = \frac{(s+1)^2(se^s + e^s) - se^s(2(s+1))}{(s+1)^4}$$

$$= \frac{\cancel{(s+1)}\left[(s+1)e^s(s+1) - 2se^s\right]}{(s+1)^{\cancel{4}\,3}}$$

$$= \frac{(s^2 + \cancel{2s} + 1 - \cancel{2s})\, e^s}{(s+1)^3}$$

$$= \frac{(s^2+1)e^s}{(s+1)^3}$$

$$g''(0) = \frac{1 \cdot e^0}{1} = \boxed{1}$$

36. $f(x) = x^2 e^x$

To determine $f^{(n)}(x)$, start taking derivatives and look for a pattern.

1) $f'(x) = \underline{2xe^x + x^2 e^x}$

2) $f''(x) = 2e^x + 2xe^x + 2xe^x + x^2e^x = \underline{2e^x + 4xe^x + x^2e^x}$

3) $f'''(x) = 2e^x + 4e^x + 4xe^x + 2xe^x + x^2e^x = \underline{6e^x + 6xe^x + x^2e^x}$

4) $f^{(4)}(x) = 6e^x + 6e^x + 6xe^x + 2xe^x + x^2e^x = \underline{12e^x + 8xe^x + x^2e^x}$

Notice the pattern:

$f^{(n)}(x) = \underline{a}e^x + \underline{b}xe^x + x^2e^x$ for some even numbers a, b.

We just need to know what a, b are in terms of n.

The coeff. of xe^x is $\underline{2n}$ (for the 1^{ST} deriv., the coeff is 2
2^{ND} deriv., the coeff is 4
etc)

The coeff. of e^x is $n(n-1)$ (for the 1^{ST} deriv: 0
2^{ND} deriv: $2 = 2 \cdot 1$
3^{RD} deriv: $6 = 3 \cdot 2$
4^{TH} deriv: $12 = 4 \cdot 3$)

So $\boxed{f^{(n)}(x) = n(n-1)e^x + 2nxe^x + x^2e^x}$

39. f - C
f' - B
f'' - A

46. $$\frac{d^2s}{dt^2} = g - \frac{0.0005}{D}\left(\frac{ds}{dt}\right)^2$$

a) $\frac{d^2s}{dt^2}$ = acceleration, $\frac{ds}{dt}$ = velocity

V_{term} = velocity when acceleration is 0

$0 = g - \frac{0.0005}{D} V_{term}^2$ so $V_{term} = \sqrt{\frac{gD}{0.0005}} = \sqrt{2000gD}$

46. b) v_{term} for raindrops of diameter 10^{-3}: $\sqrt{2000 \cdot 10^{-3} \cdot g} = \sqrt{2g} \approx \boxed{4.43 \, m/s}$

v_{term} for raindrops of diameter 10^{-4}: $\sqrt{2000 \cdot 10^{-4} g} = \sqrt{.2g} \approx \boxed{1.4 \, m/s}$

c) Raindrops accelerate more rapidly at lower velocities (a smaller number will be subtracted from g in the formula acceleration: $g - \frac{0.0005}{D}(\text{velocity})^2$)

Section 3.6

5. $\sin x \cos x$

$\frac{d}{dx} \sin x \cos x = \cos x \cdot \cos x + \sin x \, (-\sin x)$

$= \boxed{\cos^2 x - \sin^2 x}$

6. $f(x) = x^2 \cos x$

$f'(x) = 2x \cos x + x^2(-\sin x)$

$= 2x \cos x - x^2 \sin x.$

12. $k(\theta) = \theta^2 \sin^2 \theta = \theta^2 \cdot \sin\theta \cdot \sin\theta$

$k'(\theta) = 2\theta \sin^2\theta + \theta^2 \left(\frac{d}{d\theta} \sin^2\theta\right)$

$= 2\theta \sin^2\theta + \theta^2 (\sin\theta \cdot \cos\theta + \cos\theta \sin\theta)$

$= 2\theta \sin^2\theta + 2\theta^2 \sin\theta \cos\theta$

20. $f(\theta) = \theta \tan\theta \sec\theta$

$f'(\theta) = \tan\theta \sec\theta + \theta \left(\frac{d}{d\theta} [\tan\theta \sec\theta]\right)$

$= \tan\theta \sec\theta + \theta \left(\sec^2\theta \sec\theta + \tan\theta \sec\theta \tan\theta \right)$

$= \tan\theta \sec\theta + \theta \sec^3\theta + \theta \sec^2\theta \tan\theta.$

36. $\frac{d}{dx} \sec x = \frac{d}{dx} \frac{1}{\cos x} = \frac{(\cos x)(0) - 1(-\sin x)}{\cos^2 x} = \frac{\sin x}{\cos^2 x} = \boxed{\tan x \cdot \sec x}$ ✓

Section 3.7

6. $y = \cos(x^3)$

$f(u) = \cos u$

$g(x) = x^3$

Then $f \circ g(x) = \cos(x^3)$

$f'(u) = -\sin u$

$g'(x) = 3x^2$

So $f'(g(x)) = -\sin(x^3)$ and $(f \circ g)'(x) = f'(g(x))\, g'(x)$

$= -\sin(x^3) \cdot 3x^2$

$= \boxed{-3x^2 \sin(x^3)}$

43. $y = \tan(x^2+4x)$

$y' = \sec^2(x^2+4x)(2x+4) = \boxed{(2x+4)\sec^2(x^2+4x)}$

48. $y = (z+1)^4 (2z-1)^3$

$y' = 4(z+1)^3(2z-1)^3 + (z+1)^4 \cdot 3(2z-1)^2 \cdot 2$

$= \boxed{4(z+1)^3(2z-1)^3 + 6(z+1)^4(2z-1)^2}$

66 $y = \cos(te^{-2t})$

$y' = -\sin(te^{-2t}) \cdot \left(\frac{d}{dt} te^{-2t}\right)$

$= -\sin(te^{-2t})\left(e^{-2t} + t\left(\frac{de^{-2t}}{dt}\right)\right)$

$= -\sin(te^{-2t})\left(e^{-2t} + t(-2e^{-2t})\right)$

$= \boxed{-\sin(te^{-2t})\left(e^{-2t} - 2te^{-2t}\right)}$

67. $y = e^{(x^2+2x+3)^2}$

$y' = e^{(x^2+2x+3)^2} \cdot \left(2(x^2+2x+3)(2x+2)\right)$

$= \boxed{2(2x+2)(x^2+2x+3)e^{(x^2+2x+3)^2}}$

72. $y = \dfrac{1}{\sqrt{kt^4+b}} = (kt^4+b)^{-1/2}$

$y' = -\frac{1}{2}(kt^4+b)^{-3/2} \cdot (4kt^3)$

$= -2kt^3(kt^4+b)^{-3/2}$