Home work 6 (due 10/18)

Solutions

Section 3.2

25. $g(z) = 7z^{-5/14} + z^{-5} + 9$

$g'(z) = 7 \cdot (-5/14)\, z^{(-5/14 - 1)} + -5\, z^{-6} + 0$

$= \boxed{\frac{-5}{2} z^{-19/14} - 5 z^{-6}}$

28. $W(y) = 6y^4 + 7y^{2/3}$

$\boxed{W'(y) = 24y^3 + \frac{14}{3} y^{-1/3}}$

30. $f(x) = 3e^x - x^3$

$\boxed{f'(x) = 3e^x - 3x^2}$

36. $s(t) = \frac{1-2t}{t^{1/2}} = t^{-1/2} - 2t^{1/2}$

$s'(t) = \frac{-1}{2} t^{-3/2} - 2 \cdot \frac{1}{2} t^{-1/2}$

$= \boxed{\frac{-1}{2} t^{-3/2} - t^{-1/2}}$

38. $\left.\frac{dP}{dV}\right|_{V=-2}$, $P = \frac{7}{V}$

$\frac{dP}{dV} = \frac{d}{dV}(7 \cdot V^{-1}) = -7V^{-2}$

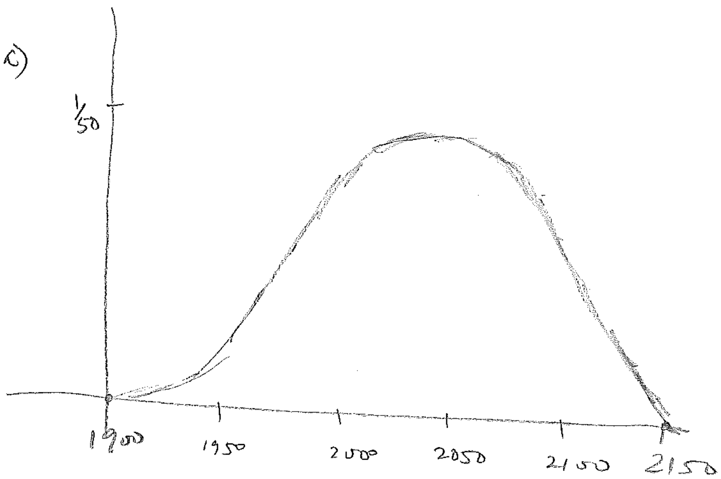
At $V = -2$, this is $-7 \cdot (-2)^{-2} = \boxed{\frac{-7}{4}}$

43. A - III
B - I
C - II
D - III

A and D have the same derivatives because their graphs are just vertical shifts of each other. Since they are the same shape, the tangent lines have the same slope (just different y-intercepts).

46. Sketch of $Q'(t)$:

a)



$Q'(t)$ represents the rate at which the oil is being produced. (barrels per year).

b) $Q'(t)$ attains its maximum in about 2000 (possibly ~2010).

c) $L = \lim_{t\to\infty} Q(t) = 2.3$ (trillion).

This number represents the maximum amount of oil that can be produced, i.e. the total amount of oil (that can be extracted) in the world.

d) $\lim_{t\to\infty} Q'(t) = 0$ since it looks like the total amount of oil produced levels off and stops increasing.

50. $f(x) = 12x - x^3$.

$f'(x) = 12 - 3x^2$

The tangent line to the graph of $f(x)$ will be horizontal when $f'(x) = 0$.

$12 - 3x^2 = 0$ when $4 - x^2 = 0$, i.e. when $x = \pm 2$.

The tangents to the graph of $f(x)$ at $(2, 16)$ and $(-2, -16)$ are horizontal.

53. Find a, b such that $p(x) = x^2 + ax + b$ satisfies

$p(1) = 0, \quad p'(1) = 4$.

$p(1) = 1 + a + b = 0$.

$p'(x) = 2x + a$, so $p'(1) = 2 + a = 4$.

So $a = 2$ and $3 + b = 0 \Rightarrow b = -3$. Thus, $\boxed{p(x) = x^2 + 2x - 3}$

59. $f(x) = xe^x$

$$f'(0) = \lim_{x\to 0} \frac{f(x+0) - f(0)}{x} = \lim_{x\to 0} \frac{f(x) - f(0)}{x} = \lim_{x\to 0} \frac{xe^x - 0}{x}$$

$$= \lim_{x\to 0} e^x = 1.$$

At $x=0$, $f(x) = 0$ and the tangent line is

$\boxed{y = x}$

60. $v_{avg} = \sqrt{\frac{8RT}{\pi M}}$

$$\frac{dv_{avg}}{dT} = \sqrt{\frac{8R}{\pi M}} \cdot \left(\frac{1}{2} T^{-\frac{1}{2}}\right)$$

At $T = 300\,K$, $\frac{dv_{avg}}{dT} = \sqrt{\frac{8R}{\pi M}} \left(\frac{1}{2\sqrt{300}}\right)$

For oxygen, $M = 0.032$ and $R = 8.31$

So $\left.\frac{dv_{avg}}{dT}\right|_{T=300} = \sqrt{\frac{8(8.31)}{\pi(0.032)}} \left(\frac{1}{2\sqrt{300}}\right) \approx \boxed{0.74}\ m/s^2$.

61. $P = 200\,m^{-1/4}$

$\frac{dP}{dm} = -50\,m^{-5/4}$, so $\left|\frac{dP}{dm}\right| = 50\,m^{-5/4}$, which is decreasing as a function of m.

For $m = 33$: $P(m) = 200(33)^{-1/4} \approx 83.45$

$P'(m) = -50\,(33)^{-5/4} \approx -0.63$

$\boxed{y - 83.45 = -0.63\,(x - 33)}$

For $m = 68$: $P(m) = 200\,(68)^{-1/4} \approx 69.65$

$P'(m) = -50\,(68)^{-5/4} \approx -0.26$

$\boxed{y - 69.65 = -0.26(x - 68)}$

66. A - III
B - I
C - II.

Section 3.3

18. $f(x) = (4e^x - x^2)(x^3+1)$

$$f'(x) = (4e^x - 2x)(x^3+1) + (4e^x - x^2)(3x^2+1)$$

20. $\left.\frac{dz}{dx}\right|_{x=-2}$, $z = \frac{x}{3x^2+1}$.

$$\frac{dz}{dx} = \frac{(3x^2+1)(1) - x(6x)}{(3x^2+1)^2} = \frac{-3x^2+1}{(3x^2+1)^2}$$

$$\left.\frac{dz}{dx}\right|_{x=-2} = \frac{-3(-2)^2+1}{(3(-2)^2+1)^2} = \frac{-12+1}{13^2} = \boxed{\frac{-11}{169}}$$

21. $f(x) = (\sqrt{x}+1)(\sqrt{x}-1)$

To compute $f'(x)$, first simplify $f(x)$.

$$f(x) = (\sqrt{x}+1)(\sqrt{x}-1) = x-1$$

So $\boxed{f'(x) = 1}$

34. $g(x) = \frac{e^{x+1}+e^x}{e+1}$.

Notice: the denominator is constant.

So $g'(x) = \frac{1}{e+1}\left(\frac{d}{dx}(e^{x+1}+e^x)\right)$

$$= \frac{1}{e+1}\frac{d}{dx}\left(e \cdot e^x + e^x\right)$$

$$= \frac{1}{e+1}\left(e \cdot \frac{d}{dx}e^x + \frac{d}{dx}e^x\right) \quad \text{(since } e \text{ is constant).}$$

$$= \frac{1}{e+1}\left[e \cdot e^x + e^x\right] = e^x\frac{(e+1)}{e+1} = \boxed{e^x}$$

38. $\frac{d}{dx}\left(\frac{ax+b}{cx+d}\right) = \frac{(cx+d)(a) - (ax+b)(c)}{(cx+d)^2}$

$= \frac{\cancel{acx} + ad - \cancel{acx} - bc}{(cx+d)^2}$

$= \boxed{\frac{ad-bc}{(cx+d)^2}}$

50. $I = \frac{V}{R}$.

a) $\frac{dI}{dR} = -VR^{-2}$

When $R=6$ and if $V=24$, $\frac{dI}{dR} = -24\cdot\left(\frac{1}{6^2}\right) = \frac{-24}{36} = \boxed{\frac{-2}{3}}$

b) $\frac{dV}{dR}$: To compute this, get V as a function of R.

$V = IR$.

Then $\frac{dV}{dR} = I$, so if $I=4$, $\left.\frac{dV}{dR}\right|_{R=6} = \boxed{4}$.

51. $R(t) = N(t)\,S(t)$.

a) $\frac{dR}{dt} = \frac{dN}{dt}S(t) + \frac{dS}{dt}N(t)$.

Given: (A) $\frac{dN}{dt} = 5$ stores/month

(B) $\frac{dS}{dt} = \$10000$/month

so $\frac{dR}{dt} = 5S(t) + 10{,}000\,N(t)$.

b) $\left.\frac{dR}{dt}\right|_{t=0} = 5S(0) + 10{,}000\,N(0)$

$= 5(150{,}000) + 10{,}000(50)$

$= 750{,}000 + 500{,}000$

$= \boxed{\$1{,}250{,}000/\text{month}}$

c) Leg A (increasing the number of stores by 5 a month) contributes more ($750,000/month) than leg B ($500,000/month)

54. $f(x) = g(x) = x$.

$$\left(\frac{f}{g}\right)' = \left(\frac{x}{x}\right)' = 1' = 0$$

$$\frac{f'}{g'} = \frac{1}{1} = 1$$

So $\left(\frac{f}{g}\right)' \neq \frac{f'}{g'}$.

55. $(f^2)' = (f \cdot f)' = f' \cdot f + f f' = 2 f f'$. ✓