Homework 3 (Due 9/27)

Solutions

Section 2.3

11. $$\lim_{x\to 2}(x+1)(3x^2-9) = \left(\lim_{x\to 2}(x+1)\right)\left(\lim_{x\to 2}(3x^2-9)\right)$$
$$= \left(\lim_{x\to 2} x + \lim_{x\to 2} 1\right)\left(\lim_{x\to 2} 3x^2 - \lim_{x\to 2} 9\right)$$
$$= (2+1)\left(3\lim_{x\to 2} x^2 - 9\right)$$
$$= 3\left(3\left(\lim_{x\to 2} x\right)^2 - 9\right) = 3\left(3\cdot 2^2 - 9\right) = 3\cdot 3 = \boxed{9}$$

18. $$\lim_{w\to 7}\frac{\sqrt{w+2}+1}{\sqrt{w-3}-1} = \frac{\lim_{w\to 7}\left(\sqrt{w+2}+1\right)}{\lim_{w\to 7}\left(\sqrt{w-3}-1\right)} = \frac{\lim_{w\to 7}\sqrt{w+2}+\lim_{w\to 7}1}{\lim_{w\to 7}\sqrt{w-3}-\lim_{w\to 7}1}$$
$$= \frac{\sqrt{\lim_{w\to 7}(w+2)}+1}{\sqrt{\lim_{w\to 7}(w-3)}-1} = \frac{\sqrt{\lim_{w\to 7}w+\lim_{w\to 7}2}+1}{\sqrt{\lim_{w\to 7}w-\lim_{w\to 7}3}-1}$$
$$= \frac{\sqrt{7+2}+1}{\sqrt{7-3}-1} = \frac{3+1}{2-1} = \boxed{4}$$

24. $$\lim_{t\to 7}\frac{(t+2)^{1/2}}{(t+1)^{2/3}} = \frac{\lim_{t\to 7}(t+2)^{1/2}}{\lim_{t\to 7}(t+1)^{2/3}} = \frac{\left(\lim_{t\to 7}(t+2)\right)^{1/2}}{\left(\lim_{t\to 7}(t+1)\right)^{2/3}}$$
$$= \frac{\left(\lim_{t\to 7}t+\lim_{t\to 7}2\right)^{1/2}}{\left(\lim_{t\to 7}t+\lim_{t\to 7}1\right)^{2/3}} = \frac{(7+2)^{1/2}}{(7+1)^{2/3}} = \frac{9^{1/2}}{8^{2/3}} = \boxed{\frac{3}{4}}$$

29. $$\lim_{x\to -4}\frac{g(x)}{x^2} = \frac{\lim_{x\to -4}g(x)}{\lim_{x\to -4}x^2} = \frac{\lim_{x\to -4}g(x)}{\left(\lim_{x\to -4}x\right)^2} = \frac{1}{(-4)^2} = \boxed{\frac{1}{16}}$$

31. The quotient law <u>cannot</u> be applied to evaluate $\lim_{x\to 0}\frac{\sin x}{x}$. One of the conditions of the quotient law is that $\lim_{x\to c}$ (denominator) is not zero. But in this case, $\lim_{x\to 0} x = 0$.

33. Give an example where $\lim_{x\to 0} (f(x)+g(x))$ exists, but neither $\lim_{x\to 0} f(x)$ nor $\lim_{x\to 0} g(x)$ exists.

There are many examples, but the simplest is $f(x)=\frac{1}{x}$, $g(x)=\frac{-1}{x}$.

$\lim_{x\to 0} \frac{1}{x}$ doesn't exist $\left(\lim_{x\to 0^-} \frac{1}{x} = -\infty, \ \lim_{x\to 0^+} \frac{1}{x} = \infty \right)$

and $\lim_{x\to 0} \frac{-1}{x}$ doesn't exist $\left(\lim_{x\to 0^-} \frac{-1}{x} = \infty, \ \lim_{x\to 0^+} \frac{-1}{x} = -\infty \right)$

but $\frac{1}{x} + \frac{-1}{x} = 0$ for all $x \neq 0$, so $\lim_{x\to 0} \left(\frac{1}{x} + \frac{-1}{x}\right) = \lim_{x\to 0} 0 = \underline{0}$

Section 2.4

12. $f(x) = \dfrac{x^2 - \cos x}{3+\cos x}$ is continuous because

1) $3+\cos x$ is not zero for any x in $\mathbb{R}$ ($-1 \leq \cos x \leq 1$, so $2 \leq 3+\cos x \leq 4$).

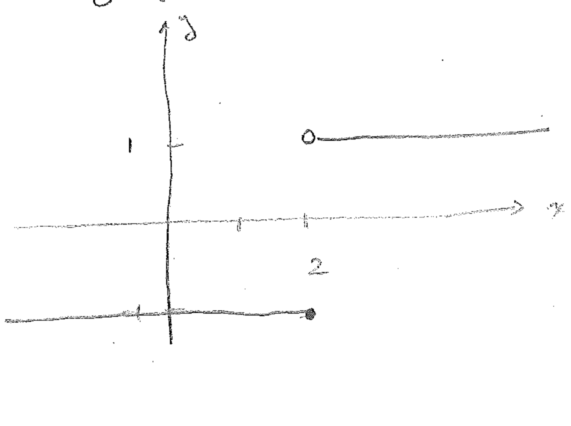
2) $x^2 - \cos x$ is continuous since it is the difference of two continuous functions (x^2 is continuous since it is a polynomial, $\cos x$ is continuous also).

3) $3+\cos x$ is continuous since it is the sum of two continuous functions (3 can be thought of as a constant polynomial).

Since quotients of continuous functions are continuous as long as the denominator isn't zero, $f(x)$ is continuous.

27. $f(x) = \begin{cases} \dfrac{x-2}{|x-2|} & x \neq 2 \\ -1 & x = 2 \end{cases}$

The graph of f looks like:



f is discontinuous at $x=2$.
It is a jump discontinuity.
The function is left continuous at $x=2$, but not right continuous at $x=2$.

(f is continuous for all other points besides $x=2$).

38. $f(x) = \dfrac{x^2}{x + x^{1/4}}$

The domain of f is all $x > 0$.

($x=0$ would make the denominator 0, so $x=0$ is not in the domain. If $x<0$, then $x^{1/4}$ is not defined. However, for $x>0$, $x + x^{1/4} \neq 0$ and $x^{1/4}$ is defined. Therefore, the domain of f is real numbers $x > 0$).

On its domain, the denominator $x + x^{1/4} \neq 0$, and both the numerator and denominator are continuous. x^2 is continuous since it's a polynomial, and $x + x^{1/4}$ is continuous because it's a sum of functions we know to be continuous. Therefore, f is continuous.

46. $f(x) = e^{-x^2}$

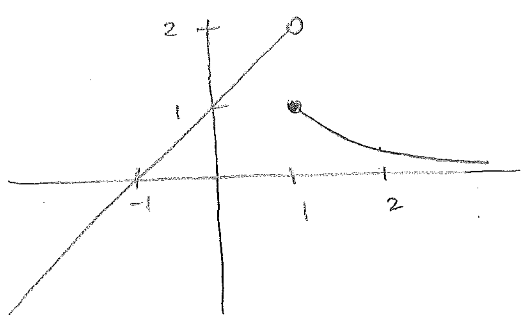
The domain of f is all real numbers.

$f(x) = g \circ h(x)$ where $g(x) = e^x$ and $h(x) = -x^2$.

Both g and h are continuous (g is exponential; h is a polynomial)

So their composition is continuous.

52. $f(x) = \begin{cases} x+1 & \text{for } x < 1 \\ \frac{1}{x} & \text{for } x \geq 1. \end{cases}$



f is discontinuous at $x=1$

f is right continuous at $x=1$, but not left continuous.

58. $f(x) = \begin{cases} 2x + 9x^{-1} & \text{for } x \leq 3 \\ -4x + c & x > 3 \end{cases}$

For f to be continuous, we need

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = f(3)$

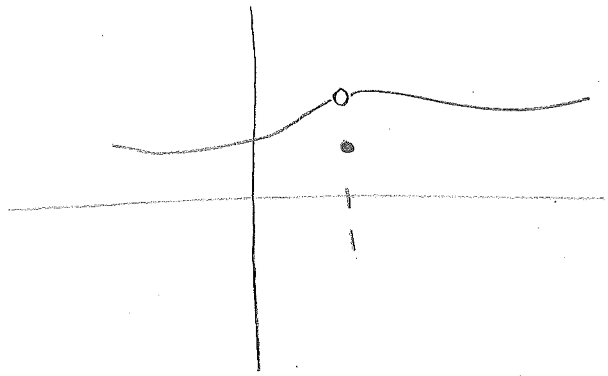
We know $f(3) = 2 \cdot 3 + 9 \cdot 3^{-1} = 6 + 3 = 9$.

Also, $\lim_{x \to 3^-} f(x) = f(3) = 9$ since $2x + 9x^{-1}$ is continuous near 3. Since $-4x+c$ is continuous near 3 also,

$\lim_{x \to 3^+} (-4x+c) = -4 \cdot 3 + c = -12 + c$.

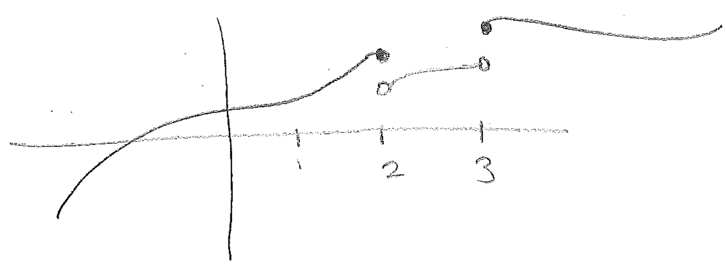
So we need $-12 + c = 9$, or $\boxed{c = 21}$ for f to be continuous

63. Want the graph of a function f such that f(x) is not continuous at 1 but $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} f(x)$.



(There are many options. Just make sure what you've drawn is the graph of a function!)

64. Want the graph of a function f such that f(x) is left continuous but not continuous at $x=2$, and f(x) is right continuous but not continuous at $x=3$.



(Again, there are many possibilities. Make sure whatever you draw is a function — ie. passes the vertical line test.)

Section 2.5

6. $\lim_{x\to 8} \dfrac{x^2-64}{x-9} = \dfrac{\lim_{x\to 8} x^2-64}{\lim_{x\to 8} x-9} = \dfrac{0}{-1} = 0$

10. $\lim_{h\to 0} \dfrac{(1+h)^3-1}{h} = \lim_{h\to 0} \dfrac{\cancel{1}+3h+3h^2+h^3-\cancel{1}}{h}$

$= \lim_{h\to 0} \dfrac{3h+3h^2+h^3}{h} = \lim_{h\to 0} (3+3h+h^2)$

$= \boxed{3}$

12. $\lim_{x\to 3} \frac{x^2-x}{x^2-9}$ If we "plug in," we get $\frac{6}{0}$, not one of the indeterminate forms.

Since the denominator is getting closer and closer to 0 while the numerator is about 6, the limit will be infinite

If x is a little bigger than 3, we have $\frac{(\text{something about } 6)}{(\text{something positive, but close to zero})}$,

which is very large (e.g. $\frac{6}{1/1000} = 6000$).

So $\boxed{\lim_{x\to 3^+} \frac{x^2-x}{x^2-9} = \infty}$

If x is a little less than 3, we have $\frac{(\text{something about } 6)}{(\text{something negative and close to } 0)}$

which is very negative (e.g. $\frac{6}{-1/1000} = -6000$).

So $\boxed{\lim_{x\to 3^-} \frac{x^2-x}{x^2-9} = -\infty}$.

30. $\lim_{x\to 1} \left(\frac{1}{1-x} - \frac{2}{1-x^2}\right) = \lim_{x\to 1} \left(\frac{1+x}{1-x^2} - \frac{2}{1-x^2}\right) = \lim_{x\to 1} \left(\frac{x-1}{1-x^2}\right)$

$= \lim_{x\to 1} \left(\frac{-1}{1+x}\right) = \frac{-1}{1+1} = \boxed{\frac{-1}{2}}$

48. $\lim_{h\to 0} \frac{(3a+h)^2-9a^2}{h} = \lim_{h\to 0} \frac{9a^2+6ah+h^2-9a^2}{h} = \lim_{h\to 0} (6a+h)$

$= \boxed{6a}$.