Homework 11 (Due 11/22)

Solutions

Section 4.2

24. $h(t) = (t^2-1)^{1/3}$

Critical points: $h'(t) = \frac{1}{3}(t^2-1)^{-2/3} \cdot 2t = \dfrac{2t}{3(t^2-1)^{2/3}}$

$h'(t)$ does not exist when $3(t^2-1)^{2/3} = 0$, i.e. when $t = \pm 1$.

$h'(t) = 0$ when the numerator, $2t = 0$, i.e. when $t = 0$.

So the critical points are $\boxed{t = 1,\ t = -1 \text{ and } t = 0}$

On $[0,1]$: Test endpoints and critical points.

$h(0) = (-1)^{1/3} = -1$

$h(1) = (0)^{1/3} = 0$

So on $[0,1]$, the maximum value is 0 (achieved when $t=1$) and the minimum value is -1 (achieved when $t=0$).

on $[0,2]$: Test endpoints and critical points.

$h(0) = -1$

$h(1) = 0$

$h(2) = (2^2-1)^{1/3} = 3^{1/3} \approx 1.44$

So on $[0,2]$, the maximum value is 1.44 (achieved when $t=2$) and the minimum value is -1 (achieved when $t=0$).

44. $y = \sqrt{1+x^2} - 2x$ on $[0,1]$.

Find critical points:

$y' = \frac{1}{2}(1+x^2)^{-1/2} \cdot (2x) - 2 = \dfrac{x}{\sqrt{1+x^2}} - 2$.

The derivative exists for all values of x, so the only critical points are from $y' = 0$.

$\dfrac{x}{\sqrt{1+x^2}} - 2 = 0$, so $\dfrac{x}{\sqrt{1+x^2}} = 2$

44 cont.

$$\frac{x}{\sqrt{1+x^2}} = 2$$

$$x = 2\sqrt{1+x^2}$$

So $x^2 = 4(1+x^2)$

$x^2 = 4 + 4x^2$ and $3x^2 = -4$. No solutions, so there are no critical points.

Test endpoints:

$y(0) = \sqrt{1} - 2\cdot 0 = 1$

$y(1) = \sqrt{2} - 2 \approx -0.59$.

So the minimum value of y on $[0,1]$ is $y(1) = -0.59$
and the maximum value of y on $[0,1]$ is $y(0) = 1$.

53. $y = \tan x - 2x$ on $[0,1]$

Find critical points:

$y' = \sec^2 x - 2$

The derivative exists for all x. Check: when is $y' = 0$?

$y' = 0 \implies \sec^2 x = 2$

$\sec x = \pm\sqrt{2}$, so $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \ldots$

$(\sec x = \pm\sqrt{2} \implies \cos x = \pm\frac{\sqrt{2}}{2})$.

The only critical point in the range $[0,1]$ is $\frac{\pi}{4}$.

Test critical points, endpoints:

$y(0) = \tan 0 - 2(0) = 0$.

$y(\frac{\pi}{4}) = \tan\frac{\pi}{4} - 2\cdot\frac{\pi}{4} = 1 - \frac{\pi}{2} \approx -0.57$

$y(1) = \tan 1 - 2 \approx -0.44$.

So the minimum value of y on $[0,1]$ is $y(\frac{\pi}{4}) \approx -0.57$
and the maximum value of y on $[0,1]$ is $y(0) = 0$.

55. $y = \frac{\ln x}{x}$ on $[1, 3]$

Find critical points.

$y = (\ln x)\left(\frac{1}{x}\right)$, so $y' = \left(\frac{1}{x}\right)\left(\frac{1}{x}\right) + (\ln x)\left(\frac{-1}{x^2}\right)$

$$= \frac{1}{x^2} + \frac{-\ln x}{x^2} = \frac{1 - \ln x}{x^2}$$

The derivative exists for all x in the domain of y, so the only critical points are when $y' = 0$.

$$\frac{1 - \ln x}{x^2} = 0 \Rightarrow 1 - \ln x = 0$$

So $\ln x = 1$ and $x = e$.

Test critical points and end points.

$y(1) = \frac{\ln 1}{1} = 0$

$y(e) = \frac{\ln e}{e} = \frac{1}{e} \approx 0.368$.

$y(3) = \frac{\ln 3}{3} \approx 0.366$

So the maximum value of y on $[1, 3]$ is $y(e) \approx 0.368$. and the minimum value of y on $[1, 3]$ is $y(1) = 0$.

71. Prove that $f(x) = x^4 + 5x^3 + 4x$ has no root c satisfying $c > 0$.

First, notice that $f(0) = 0 + 5 \cdot 0 + 4 \cdot 0 = 0$, so 0 is a root of f.

Also, $f'(x) = 4x^3 + 15x^2 + 4$.

For any $x > 0$, $f'(x) > 0$.

If f had a root c with $c > 0$, then $f(c) = 0 = f(0)$, and Rolle's theorem says then that there is some point a in $(0, c)$ with $f'(a) = 0$. But if $0 < a < c$, then in particular $a > 0$ and $f'(a) = 4a^3 + 15a^2 + 4 > 0$. So there can't be a root c with $c > 0$.

77. $P = \frac{1}{2}mv_1^2 - \frac{1}{2}mv_2^2$ watts, where $m = \rho A \frac{(v_1+v_2)}{2}$

$P_0 = \frac{1}{2}\rho A v_1^3$ and $F = \frac{P}{P_0}$.

a) $$F = \frac{\frac{1}{2}mv_1^2 - \frac{1}{2}mv_2^2}{\frac{1}{2}\rho A v_1^3} = \frac{\cancel{\frac{1}{2}} m(v_1^2 - v_2^2)}{\cancel{\frac{1}{2}}\rho A v_1^3} = \frac{\cancel{\rho}\cancel{A}\frac{(v_1+v_2)}{2}(v_1^2-v_2^2)}{\cancel{\rho}\cancel{A}v_1^3}$$

$$= \frac{1}{2}\frac{(v_1+v_2)(v_1^2-v_2^2)}{(v_1)\cdot(v_1^2)} = \frac{1}{2}\left(\frac{v_1+v_2}{v_1}\right)\left(\frac{v_1^2-v_2^2}{v_1^2}\right)$$

$$= \frac{1}{2}\left(1+\frac{v_2}{v_1}\right)\left(1-\frac{v_2^2}{v_1^2}\right) = \boxed{\frac{1}{2}(1+r)(1-r^2)}$$ ✓

Note that $0 \le r$ since $v_2 \ge 0$ (wind speed must be non-negative)

and $r \le 1$ since $v_2 \le v_1$ (wind speed exiting the turbine can't be more than wind speed entering)

b) Maximize $F(r)$:

Critical points: $F'(r) = \frac{1}{2}\left[(1+r)(-2r) + (1-r^2)\right]$

$= \frac{1}{2}(-2r - 2r^2 + 1 - r^2)$

$= -\frac{3}{2}r^2 - r + \frac{1}{2}$ exists for all r, so check when $F'(r) = 0$.

$-\frac{3}{2}r^2 - r + \frac{1}{2} = 0 \Rightarrow 3r^2 + 2r - 1 = 0$.

$(3r-1)(r+1) = 0$, so $r = \frac{1}{3}$ or $r = -1$

Test critical points (in the range $[0,1]$) and end points:

$F(0) = \frac{1}{2}(1+0)(1-0) = \frac{1}{2}$

$F(\frac{1}{3}) = \frac{1}{2}(\frac{4}{3})(1-\frac{1}{9}) = \frac{2}{3}\cdot\frac{8}{9} = \frac{16}{27}$

$F(1) = \frac{1}{2}(1+1)(1-1) = 0$.

So the maximum value of $F(r)$ is $\boxed{\frac{16}{27}}$ ✓

c) Betz's formula for $F(r)$ is not meaningful for r near zero because if $r=0$, $v_2=0$ and no air is leaving the turbine. This doesn't make sense. Some air must leave the turbine.

81. Maximize:
$y = x^a - x^b$ on $[0,1]$ $(0 < a < b)$.

Find critical points:

$$y' = a x^{a-1} - b x^{b-1} = 0.$$

$$a x^{a-1} = b x^{b-1}$$

$$\frac{a}{b} = \frac{x^{b-1}}{x^{a-1}} = x^{b-a}$$

So $x = \left(\frac{a}{b}\right)^{\frac{1}{b-a}}$

Test critical points and end points.

$y(0) = 0$

$y(1) = 0$

$$y\left(\left(\tfrac{a}{b}\right)^{\frac{1}{b-a}}\right) = \left(\frac{a}{b}\right)^{\frac{a}{b-a}} - \left(\frac{a}{b}\right)^{\frac{b}{b-a}} = \left(\frac{a}{b}\right)^{\frac{a}{b-a}}\left(1 - \left(\frac{a}{b}\right)^{\frac{b-a}{b-a}}\right)$$

$$= \boxed{\left(\frac{a}{b}\right)^{\frac{a}{b-a}}\left(1 - \left(\frac{a}{b}\right)\right)}$$

is the maximum.

For $y = x^5 - x^{10}$, maximum is

$$\left(\frac{5}{10}\right)^{\frac{5}{10-5}}\left(1 - \left(\frac{5}{10}\right)\right) = \left(\frac{1}{2}\right)^1\left(1 - \frac{1}{2}\right) = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \boxed{\frac{1}{4}}.$$

90. Given: $\theta = 4r - 2i$ and Snell's Law $\sin i = n \sin r$ $(n \approx 1.33)$.

a) Differentiate $\sin i = n \sin r$ with respect to i.

$$\cos i = n \cos r \cdot \frac{dr}{di}.$$

So $$\frac{dr}{di} = \frac{\cos i}{n \cos r}.$$

90 continued

b) To maximize θ, differentiate $\theta = 4r - 2i$

$$\frac{d\theta}{di} = 4\frac{dr}{di} - 2 = 4\frac{\cos i}{n\cos r} - 2.$$

So $\frac{d\theta}{di} = 0$ when $\frac{4\cos i}{n\cos r} - 2 = 0$

i.e when $\cos i = \frac{2n\cos r}{4} = \frac{n\cos r}{2}$.

Snell's law says $\sin i = n\sin r$, so $\sin^2 i = n^2\sin^2 r = n^2(1-\cos^2 r)$

so $1-\cos^2 i = n^2 - n^2\cos^2 r$ ①

and $\cos i = \frac{n}{2}\cos r \Rightarrow \cos^2 i = \frac{n^2}{4}\cos^2 r$.

So $4\cos^2 i = n^2\cos^2 r$. ②

Combining ① and ②, $1-\cos^2 i = n^2 - 4\cos^2 i$

So $1 + 3\cos^2 i = n^2$ and $\boxed{\cos i = \sqrt{\frac{n^2-1}{3}}}$ ✓

c) Since $\cos i = \sqrt{\frac{n^2-1}{3}} = \sqrt{\frac{(1.33)^2-1}{3}}$, then $i = \arccos\sqrt{\frac{1.33^2-1}{3}} \approx 59.58°$

So use this to find r.

$\sin 59.58° = 1.33\sin r$, so $r = \arcsin\left(\frac{\sin 59.58°}{1.33}\right) \approx 40.42°$

Thus $\theta_{max} = 4r - 2i \approx \boxed{42.53°}$

92. $f(x) = x^2 - 2x + 3$.

To find the minimum, find critical points.

$f'(x) = 2x - 2 = 0 \implies x = 1$.

There is a minimum at $x=1$ since $\lim_{x \to -\infty} f(x) = \lim_{x \to \infty} f(x) = \infty$.

So the minimum value of $f(x)$ is $f(1) = 1 - 2 + 3 = \boxed{2}$

But then $f(x) \geq 2$ for all x, and $f(x) = x^2 - 2x + 3$ takes on only positive values.

For $f(x) = x^2 + rx + s$:

Find critical points

$$f'(x) = 2x + r = 0 \implies x = \frac{-r}{2}.$$

Again, this critical point is ~~a~~ the location of a minimum since $\lim_{x \to \infty} f(x) = \lim_{x \to -\infty} f(x) = \infty$.

So the minimum value of $f(x)$ is

$$f\left(\frac{-r}{2}\right) = \left(\frac{-r}{2}\right)^2 + r\left(\frac{-r}{2}\right) + s$$

$$= \frac{r^2}{4} - \frac{r^2}{2} + s = \frac{-r^2}{4} + s$$

If the minimum value is greater than 0, then $f(x) > 0$ for all x and f takes on only positive values.

So if $\boxed{\frac{-r^2}{4} + s > 0}$, $f(x)$ takes on only positive values.

For $r = 0$, $s = -1$, $f(x) = x^2 - 1$ takes on both positive and negative values.

Section 4.7.

4. Problem says: Find x such that $x + \frac{1}{x}$ is minimized (for x positive).

Let $f(x) = x + \frac{1}{x}$.

Then we're optimizing f over the interval $(0, \infty)$, which is open.

Find critical points:

$$f'(x) = 1 + \frac{-1}{x^2} = 0 \quad \text{when} \quad \frac{1}{x^2} = 1, \text{ i.e. } x = \pm 1.$$

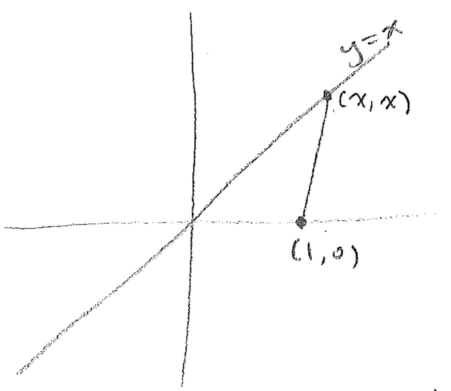
The only critical point in our interval is $x = 1$.

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} x + \frac{1}{x} = \infty \quad \text{and} \quad \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} x + \frac{1}{x} = \infty.$$

So $f(1) = 2$ is the minimum value of f.

$\boxed{x = 1}$ minimizes $x + \frac{1}{x}$.

11. Find the point on $y = x$ closest to $(1, 0)$.



Distance from $(1, 0)$ to a point (x, x) on $y = x$ is $\sqrt{(x-1)^2 + (x-0)^2}$

So distance squared is

$$D = (x-1)^2 + x^2.$$

Minimize D to minimize distance.

$$D' = 2(x-1) + 2x = 0$$

$$2x + 2x - 2 = 0$$

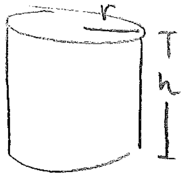
$$4x = 2 \Rightarrow x = \frac{1}{2}.$$

The only critical point is $x = \frac{1}{2}$, and x is in $(-\infty, \infty)$.

$\lim_{x \to -\infty} D = \infty$ and $\lim_{x \to \infty} D = \infty$, so this critical point is the location of a minimum.

Thus, $\boxed{\left(\frac{1}{2}, \frac{1}{2}\right)}$ is the point on $y = x$ closest to $(1, 0)$.

18.



Maximize volume.

Surface area = A fixed.

$V = \pi r^2 h$

$A = 2\pi r^2 + 2\pi r h$

So $h = \dfrac{A - 2\pi r^2}{2\pi r}$

$$V(r) = \cancel{\pi} r^2 \left(\frac{A - 2\pi r^2}{2\cancel{\pi} \cancel{r}} \right) = \frac{1}{2} r (A - 2\pi r^2) = \frac{1}{2}(Ar - 2\pi r^3)$$

Find critical points: $V'(r) = \frac{1}{2}(A - 6\pi r^2) = 0$

$A = 6\pi r^2$, so $r = \sqrt{\frac{A}{6\pi}}$ is the only critical point that makes sense ($r > 0$).

Our interval is $r \in (0, \infty)$, which is open.

$$\lim_{r \to 0} V(r) = \lim_{r \to 0} \frac{1}{2}(Ar - 2\pi r^3) = 0.$$

$$\lim_{r \to \infty} V(r) = \lim_{r \to \infty} \frac{1}{2}(Ar - 2\pi r^3) = -\infty$$

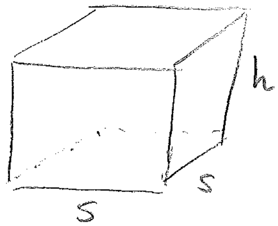
So at $r = \sqrt{\frac{A}{6\pi}}$, V has a maximum.

Volume is maximized when $r = \sqrt{\frac{A}{6\pi}}$ and $h = \dfrac{A - 2\pi\left(\frac{A}{\cancel{6}_3\cancel{\pi}}\right)}{2\pi\sqrt{\frac{A}{6\pi}}}$

i.e. when $h = \dfrac{\frac{2}{3}A}{2\pi\sqrt{\frac{A}{6\pi}}} = \dfrac{A\sqrt{6\pi}}{3\pi\sqrt{A}} = \dfrac{\sqrt{A}\sqrt{2}}{\sqrt{\pi}\sqrt{3}} = \sqrt{\dfrac{2A}{3\pi}}$

So $\boxed{r = \sqrt{\frac{A}{6\pi}}, \quad h = \sqrt{\frac{2A}{3\pi}}}$

20.



$4s + h = 108$ in

Maximize volume.

$V = s^2 h$ and $h = 108 - 4s$.

So $V(s) = s^2(108-4s) = 108s^2 - 4s^3$.

Find critical points: $V'(s) = 216s - 12s^2 = 0$

$12s(18-s) = 0$.

So $s = 0$ or $s = 18$.

But the interval for s is $(0, \infty)$, so the only critical point in our interval is $s = 18$.

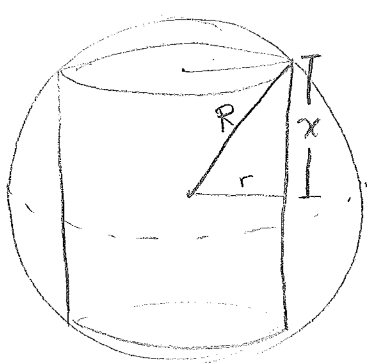
Check: $\lim_{s \to 0^+} V(s) = 0$ and $\lim_{s \to \infty} V(s) = -\infty$, so the critical point is the location of a maximum.

So the dimensions that maximize volume are

s = side length of base = $\boxed{18 \text{ in}}$

h = height $= 108 - 4(18) = 108 - 72 = \boxed{36 \text{ in}}$

21.



$V = \pi r^2 h = \pi r^2 (2x)$

But $r^2 + x^2 = R^2$ (where R = radius of sphere)

So $r^2 = R^2 - x^2$.

So $V = 2\pi x(R^2 - x^2) = 2\pi(R^2 x - x^3)$

Find critical points: $V'(x) = 2\pi(R^2 - 3x^2) = 0$

$V'(x) = 0$ when $R^2 = 3x^2$, i.e. when $x = \frac{\pm R}{\sqrt{3}}$

But the interval for x is $(0, R)$, so choose only $x = \frac{R}{\sqrt{3}}$.

$\lim_{x \to 0} V = 0$ and $\lim_{x \to R} V = 0$, so $x = \frac{R}{\sqrt{3}}$ is the location of a maximum.

Volume is maximized when the $\boxed{\text{height is } \frac{2R}{\sqrt{3}} \text{ and radius is } \frac{\sqrt{2}R}{\sqrt{3}}}$

39. 100 units in the apartment building, all rented when rent is \$900.
One unit becomes vacant for every \$10 in rent
Each occupied unit costs \$80 in maintainence

Profit = P = (# units occupied) (rent) − 80 (# units occupied).

rent = $900 + 10x$ and at this rate # units occupied = $100 - x$.

So $P = (100-x)(900+10x) - 80(100-x)$

$= 90{,}000 + 1000x - 900x - 10x^2 - 8000 + 80x$

$= 82{,}000 + 180x - 10x^2$

Maximize P over the interval $[0, 100]$ ($x \geq 0$ since we're raising the rent. $x \leq 100$ since we can't rent out more than 100 units).

Critical points: $P' = 180 - 20x = 0$

$x = \frac{180}{20} = 9.$

Test critical points, end points:

$P(9) = 82000 + 180(9) - 10(81)$

$= 82000 + 1620 - 810$

$= 82810$

$P(0) = 8200$

$P(100) = 0.$

So the maximum profit is \$82,810 and occurs when the rent is $\boxed{\$990}$

41. $P = 2LK^2$ where L = cost of labor, K = cost of equipment.

Want: $P = 1.7$ million units/month.

and want to minimize Cost = L+K.

$C = L + K$ and $1.7 = 2LK^2$.

So $L = \frac{1.7}{2K^2}$ and $C(K) = K + \frac{1.7}{2K^2}$.

So the problem is to minimize $C(K)$ over the interval $K \in (0, \infty)$.

41. cont.

Critical points: $C'(K) = 1 + \frac{1.7}{\cancel{2}}(\cancel{-2})K^{-3} = 1 - \frac{1.7}{K^3} = 0$

$C'(K) = 0$ when $K^3 = 1.7$

i.e. $K \approx 1.19$.

$$\lim_{K\to 0+} C(K) = \lim_{K\to 0+} K + \frac{1.7}{2K^2} = \infty$$

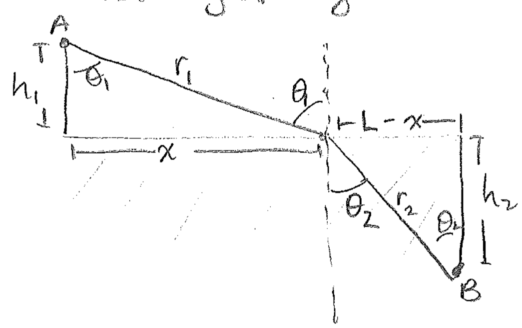
$$\lim_{K\to \infty} C(K) = \lim_{K\to \infty} K + \frac{1.7}{2K^2} = \infty.$$

So the critical point $K \approx 1.19$ is a minimum.

Cost is minimized when $\boxed{\begin{array}{l} K = 1.19 \text{ million euros and} \\ L = \frac{1.7}{2K^2} = 0.60 \text{ million euros} \end{array}}$

44. v_1 = velocity of light in air

v_2 = velocity of light in water.



Time $= T = \frac{r_1}{v_1} + \frac{r_2}{v_2}$.

and $r_1 = \sqrt{h_1^2 + x^2}$

$r_2 = \sqrt{h_2^2 + (L-x)^2}$

(where h_1, h_2, L are fixed).

$$\text{So } T = \frac{\sqrt{h_1^2 + x^2}}{v_1} + \frac{\sqrt{h_2^2 + (L-x)^2}}{v_2}.$$

Minimize T over $[0, L]$.

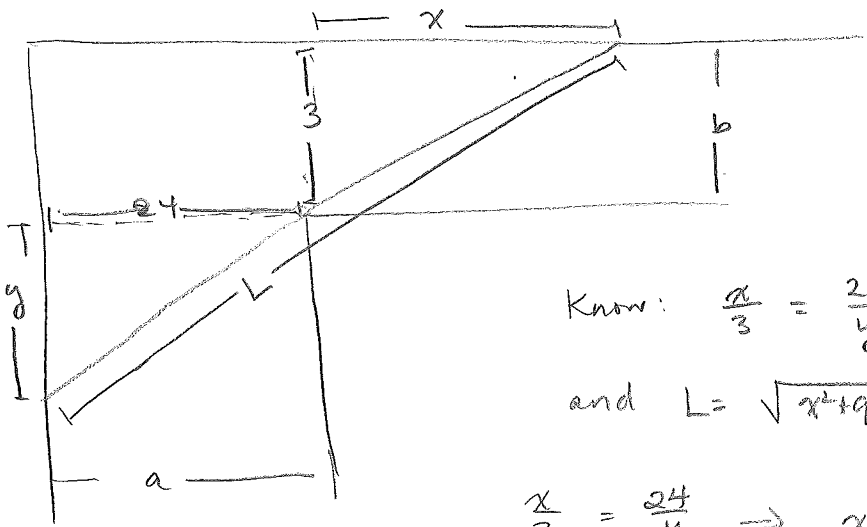
Find critical points: $T' = \frac{1}{2v_1}(h_1^2 + x^2)^{-\frac{1}{2}} \cancel{2}x + \frac{1}{2v_2}(h_2^2 + (L-x)^2)^{-\frac{1}{2}}(-2(L-x))$

$$= \frac{x}{v_1\sqrt{h_1^2 + x^2}} - \frac{L-x}{v_2\sqrt{h_2^2 + (L-x)^2}}$$

So $T' = 0$ when $\frac{x}{v_1\sqrt{h_1^2 + x^2}} = \frac{L-x}{v_2\sqrt{h_2^2 + (L-x)^2}}$

i.e. when $\frac{x}{v_1 r_1} = \frac{L-x}{v_2 r_2}$, that is $\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$. ✓

59.



Know: $\frac{x}{3} = \frac{24}{y}$ using similar triangles

and $L = \sqrt{x^2+9} + \sqrt{24^2+y^2}$

$$\frac{x}{3} = \frac{24}{y} \Rightarrow xy = 72, \text{ so } y = \frac{72}{x}$$

$$L = \sqrt{x^2+9} + \sqrt{24^2+\left(\frac{72}{x}\right)^2}$$

The $\underline{\text{max}}$ length of the pole is equal to the $\underline{\text{minimum}}$ diagonal distance L.

So minimize L over x in $(0, \infty)$

Critical points: $$L' = \frac{1}{\cancel{2}}(x^2+9)^{-\frac{1}{2}} \cdot \cancel{2}x + \frac{1}{\cancel{2}}\left(24^2+\left(\frac{72}{x}\right)^2\right)^{-\frac{1}{2}} \cdot \cancel{2}\left(\frac{72}{x}\right) \cdot \left(-\frac{72}{x^2}\right)$$

$$= \frac{x}{\sqrt{x^2+9}} - \frac{72^2}{x^3\sqrt{24^2+\left(\frac{72}{x}\right)^2}}$$

$$= \frac{x}{\sqrt{x^2+9}} - \frac{72^2}{x^2\sqrt{24^2x^2+72^2}}$$

$$= \frac{x}{\sqrt{x^2+9}} - \frac{72^2}{24x^2\sqrt{x^2+9}} = \frac{24x^3-72^2}{24x^2\sqrt{x^2+9}}$$

$L' = 0$ when $24x^3 - 72^2 = 0$

ie $x^3 = \frac{72^2}{24} = 216$

$\underline{x = 6}$

$$\lim_{x\to 0} L = \lim_{x\to 0}\Big(\underbrace{\sqrt{x^2+9}}_{\to 3} + \underbrace{\sqrt{24^2+\left(\tfrac{72}{x}\right)^2}}_{\to\infty}\Big) = \infty$$

and $$\lim_{x\to\infty} L = \lim_{x\to\infty}\Big(\underbrace{\sqrt{x^2+9}}_{\to\infty} + \underbrace{\sqrt{24^2+\left(\tfrac{72}{x}\right)^2}}_{\to 24}\Big) = \infty.$$

So L is minimized when $x = 6$.

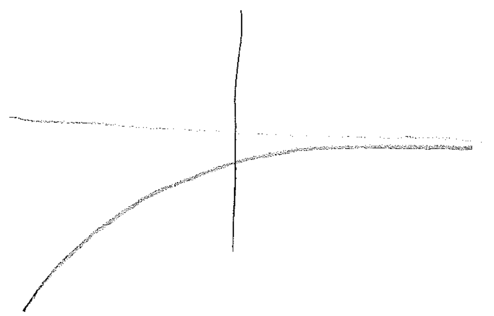
So the max length of the pole is $\sqrt{6^2+9} + \sqrt{24^2+12^2} = 3\sqrt{5}+12\sqrt{5} = \boxed{15\sqrt{5}}$

Section 4.3

Preliminary questions

2. (c) Does not follow from MVT.
f could have a tangent line with slope 0, but no secant line with slope 0 (e.g. $y = x^3$).

3. Yes, a function can take only negative values, but have positive derivative.



Exercises.

8. $y = e^x - x$, $[-1, 1]$.

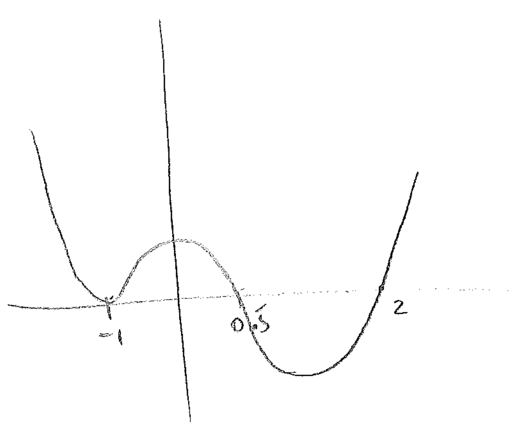
$$\frac{y(1) - y(-1)}{1-(-1)} = \frac{(e-1) - (\frac{1}{e}+1)}{2} = \frac{e - \frac{1}{e}}{2}$$

$$y' = e^x - 1 = \frac{e - \frac{1}{e} - 2}{2}$$

$$e^x = \frac{e - \frac{1}{e} - 2}{2} + \frac{2}{2} = \frac{e - \frac{1}{e}}{2}$$

$$x = \ln\left(\frac{e - \frac{1}{e}}{2}\right) \approx 0.897$$

14. $y = f'(x)$



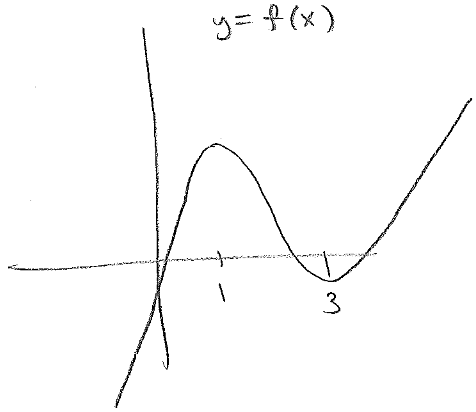
Critical points of $f(x)$:

$x = -1 \to$ Neither a local max. nor min.

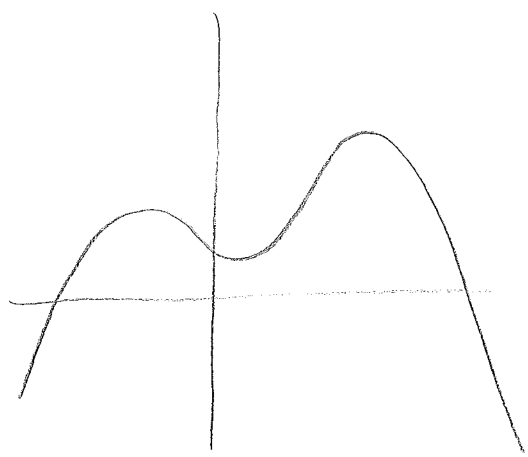
$x = 0.5 \to$ Local maximum (f' switches from $+$ to $-$)

$x = 2 \to$ Local minimum (f' switches from $-$ to $+$).

17. $f'(x)$ is negative on $(1,3)$, positive everywhere else



18. $f'(x)$ makes sign transitions $+, -; +, -$



58. a) "Avg. velocity was 70 mph, but speedometer never read 70 mph" contradicts the Mean Value Theorem.

b) "Clocked going 70mph, but speedometer never read 65 mph" contradicts the Intermediate Value Theorem.