Homework 1 (Due 9/13)

Solutions

Section 1.1

44. $g(t) = \sqrt{2-t}$

Domain: We need $2-t \geq 0$ because we can't take the square root of a negative number.

So the domain is $\boxed{\text{real numbers } t \text{ such that } t \leq 2}$

(also written $(-\infty, 2]$ or $\{t \mid t \in \mathbb{R},\ t \leq 2\}$)

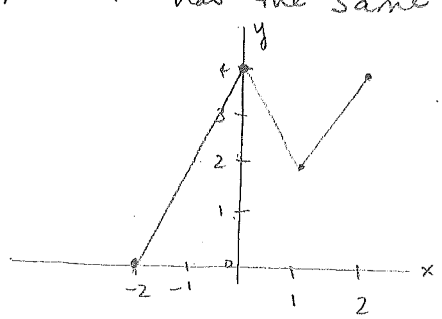
Range: When unspecified, square roots are non-negative, so the range is $\boxed{g(t) \geq 0}$

47. $f(x) = \frac{1}{x^2}$

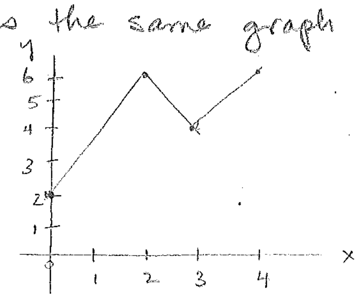
Domain: The only thing that could go wrong here is dividing by 0, so the domain is $\boxed{\text{real numbers not equal to } 0}$

Range: The only value we can't get is 0, so the range is $\boxed{(-\infty, 0) \cup (0, \infty)}$

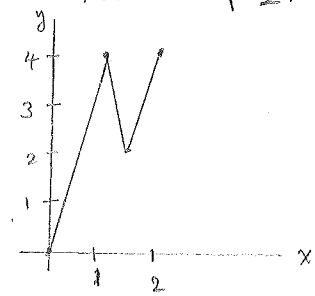
66. $f(x+2)$ has the same graph as $f(x)$, but shifted left by 2.



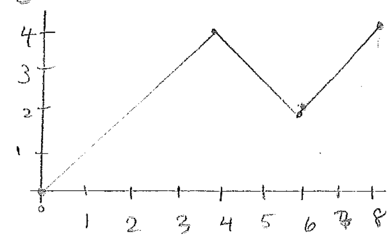
$f(x)+2$ has the same graph as $f(x)$ but shifted up by 2.



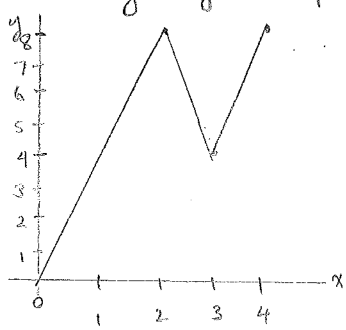
67. The graph of $f(2x)$ has the same graph as $f(x)$ but compressed horizontally by a factor of 2.



The graph of $f(\frac{1}{2}x)$ is the same as the graph of $f(x)$, but stretched horizontally by a factor of 2.



The graph of $2f(x)$ is the same as the graph of $f(x)$, but stretched vertically by a factor of 2.



Section 1.2

11. Slope 3, passing through $(7,9)$

Use slope-intercept form.

$\boxed{y-9 = 3(x-7)}$ or $y-9 = 3x-21$

so $\boxed{y = 3x-12}$

14. Passing through $(-1, 4)$ and $(2, 7)$

$$\text{Slope} = \frac{7-4}{2-(-1)} = \frac{3}{3} = 1$$

Using slope-intercept form, get $\boxed{y-4 = x+1}$

or $\boxed{y = x+5}$

20. Slope 3, x-intercept 6.

x-intercept 6 means the line passes through $(6,0)$

Use slope intercept form:

$$y-0 = 3(x-6) \quad \text{or} \quad \boxed{y = 3x-18}$$

53. If $f(x)$ and $g(x)$ are linear, then so is $f(x)+g(x)$.

If $f(x)$ and $g(x)$ are linear, $f(x) = mx+b$ for some real numbers m, b
and $g(x) = nx+c$ for some real numbers n, c.

Then $f(x)+g(x) = mx+b+nx+c$

$= (m+n)x + (b+c)$

Therefore $f(x)+g(x)$ is linear (it is of the form $Ax+B$ where A and B are real numbers).

However, $f(x)g(x)$ is NOT linear. It is quadratic.

$(f(x)g(x) = (mx+b)(nx+c) = mnx^2 + (mc+nb)x + bc)$

Section 1.3

29. $f(x) = 2^x$, $g(x) = x^2$

$f\circ g(x) = f(x^2) = 2^{x^2}$

Domain: all real numbers.

$g\circ f(x) = g(2^x) = (2^x)^2 = 2^{2x}$

Domain: all real numbers.

32. $f(x) = \frac{1}{x^2+1}$, $g(x) = x^{-2}$

$f\circ g(x) = f(x^{-2}) = \frac{1}{(x^{-2})^2+1} = \frac{1}{x^{-4}+1} = \frac{1}{\frac{1}{x^4}+1}$

Domain: all real numbers not equal to 0.

$g\circ f(x) = g\left(\frac{1}{x^2+1}\right) = \left(\frac{1}{x^2+1}\right)^{-2} = (x^2+1)^2$

Domain: all real numbers.

33. $f(t) = \frac{1}{\sqrt{t}}$, $g(t) = -t^2$

$f \circ g(t) = f(-t^2) = \frac{1}{\sqrt{-t^2}}$

Domain: The domain is empty; there are no possible inputs.

$g \circ f(t) = g\left(\frac{1}{\sqrt{t}}\right) = -\left(\frac{1}{\sqrt{t}}\right)^2 = -\frac{1}{t}$

Domain: all real numbers not equal to 0.

34. $f(t) = \sqrt{t}$, $g(t) = 1 - t^3$

$f \circ g(t) = f(1-t^3) = \sqrt{1-t^3}$

Domain: We need $1 - t^3 \geq 0$, or $t \leq 1$

$g \circ f(t) = g(\sqrt{t}) = 1 - (\sqrt{t})^3 = 1 - t^{3/2}$

Domain: $t \geq 0$ (we can't take the square root of a negative number).

Section 1.4

3. a) $1 \text{ radian} \times \frac{180 \text{ degrees}}{\pi \text{ radians}} = \boxed{\frac{180}{\pi} \text{ degrees}}$

b) $\frac{\pi}{3} \text{ radians} \times \frac{180 \text{ degrees}}{\pi \text{ radians}} = \frac{180}{3} = \boxed{60 \text{ degrees}}$

c) $\frac{5}{12} \text{ radians} \times \frac{180 \text{ degrees}}{\pi \text{ radians}} = \boxed{\frac{75}{\pi} \text{ degrees}}$

d) $-\frac{3\pi}{4} \text{ radians} \times \frac{180 \text{ degrees}}{\pi \text{ radians}} = \boxed{-135 \text{ degrees}}$

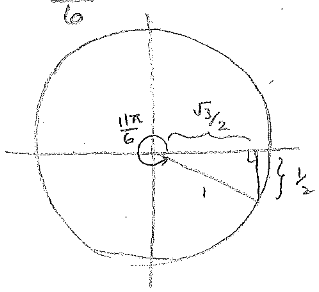
4. a) $1 \text{ degree} \times \frac{\pi \text{ radians}}{180 \text{ degrees}} = \boxed{\frac{\pi}{180} \text{ radians}}$

b) $30^\circ \times \frac{\pi \text{ radians}}{180^\circ} = \boxed{\frac{\pi}{6} \text{ radians}}$

c) $25^\circ \times \frac{\pi \text{ radians}}{180^\circ} = \boxed{\frac{5\pi}{36} \text{ radians}}$

d) $120^\circ \times \frac{\pi \text{ radians}}{180^\circ} = \boxed{\frac{2\pi}{3} \text{ radians}}$

8. $\theta = \frac{11\pi}{6}$



$\sin \frac{11\pi}{6} = -\frac{1}{2}$

$\cos \frac{11\pi}{6} = \frac{\sqrt{3}}{2}$

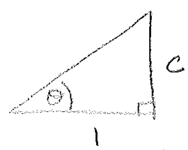
$\tan \frac{11\pi}{6} = \frac{\sin \frac{11\pi}{6}}{\cos \frac{11\pi}{6}} = \frac{-\frac{1}{2}}{\sqrt{3}/2} = \frac{-1}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$

$\sec \frac{11\pi}{6} = \frac{1}{\cos \frac{11\pi}{6}} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2\sqrt{3}}{3}$

$\cot \frac{11\pi}{6} = \frac{\cos \frac{11\pi}{6}}{\sin \frac{11\pi}{6}} = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3}$

$\csc \frac{11\pi}{6} = \frac{1}{\sin \frac{11\pi}{6}} = \frac{1}{-\frac{1}{2}} = -2$

17. $\tan\theta = c, \quad 0 \le \theta < \frac{\pi}{2}.$



$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{c}{1}$

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{1}{?}$

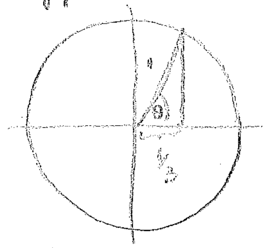
Use the Pythagorean theorem to find the hypotenuse.

$x^2 = 1^2 + c^2 \implies x = \sqrt{1+c^2}$

So $\cos\theta = \frac{1}{\sqrt{1+c^2}}$, as desired.

18. Given $\cos\theta = \frac{1}{3}$.

a) Suppose $0 \le \theta < \frac{\pi}{2}$.



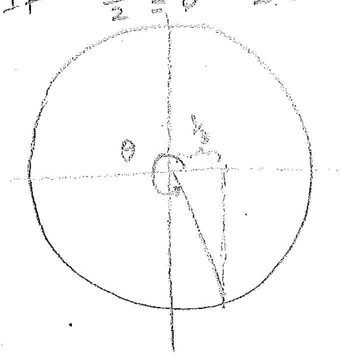
Use the Pythagorean theorem.

$\left(\frac{1}{3}\right)^2 + \sin^2\theta = 1$

$\sin^2\theta = \frac{8}{9}$, so $\boxed{\sin\theta = \frac{2\sqrt{2}}{3}}$

For $\tan\theta$, use $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{2\sqrt{2}}{3}}{\frac{1}{3}} = \boxed{2\sqrt{2}}$ ✓

b) If $\frac{3\pi}{2} \le \theta < 2\pi$.

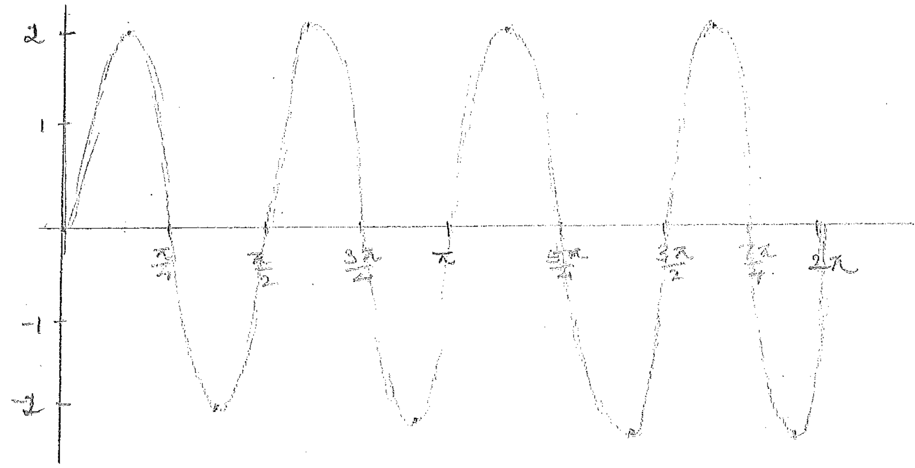


This is the same triangle as in part a, but flipped down across the x-axis.

So $\sin\theta = -\frac{2\sqrt{2}}{3}$

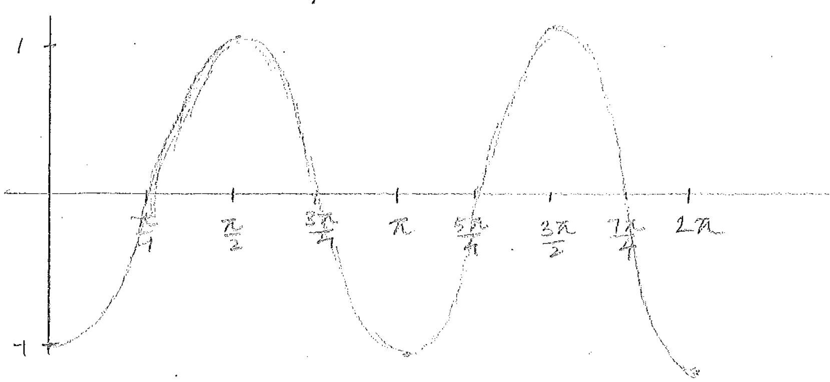
$\tan\theta = -2\sqrt{2}$.

35. $2\sin 4\theta$



(compressed horizontally by a factor of 4, stretched vertically by a factor of 2).

36. $\cos\left(2\left(\theta - \frac{\pi}{2}\right)\right)$



(compressed horizontally by a factor of 2, shifted right by $\frac{\pi}{2}$).