* Elimination matrices are the identity matrix $\begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & \ddots \; 1 \end{bmatrix}$ with a multiplier l in the ij position (i^{th} row, j^{th} column)

Example: $\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ will subtract 2 times row 1 from row 2.

$$\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 4 & -2 \\ 4 & 9 & -3 \\ -2 & -3 & 7 \end{bmatrix} = \begin{bmatrix} 2 & 4 & -2 \\ 0 & 1 & 1 \\ -2 & -3 & 7 \end{bmatrix}$$

* Permutation matrices are the identity matrix with rows switched.

P_{ij} switches rows i and j (and the matrix is I with rows i and j switched).

E.g. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ switches rows 2 and 3.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 4 & -2 \\ 0 & 0 & 1 \\ 0 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 4 & -2 \\ 0 & 2 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

Class 3: 9/13/12

Outline

I. Elimination: pathological cases

II. Elimination with matrices.

III. Matrix operations + rules.

Announcements

Collect HW 1

Registration: Stay after to fill out form.

QUESTIONS?

Last time: Triangular systems are easy to solve. Elimination: goal: make things triangular.

Key fact: Elimination works when you get as many pivots as equations.

Go back to prev. day's notes: Infinitely many solns.

Laws of Matrix operations

Addition is commutative $A+B = B+A$

associative $(A+B)+C = A+(B+C)$

Scalar mult. distributes over addition

$$c(A+B) = cA + cB$$

Matrix mult. distributes over addition

$$A(B+C) = AB + AC$$

$$(B+C)A = BA + CA$$

Matrix mult is associative $(AB)C = A(BC)$

WARNING: $AB \neq BA$ except in very special cases.

Ex: $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

but

$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Non ex: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Note: $I = \begin{bmatrix} 1 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & 1 \end{bmatrix}$ is called the identity matrix.
(there is one for each size n)

$$AI = IA = A$$

Definition: If A is a square matrix, its inverse is the matrix A^{-1} so that

$$AA^{-1} = A^{-1}A = I. \quad \text{(if such a matrix exists)}$$

If A^{-1} exists, A is invertible. If not, A is not invertible.

Facts:
- An inverse exists exactly when (if and only if) elimination gives a full set of pivots.
- If A is invertible, the only solution to $A\vec{x} = \vec{b}$ is $\vec{x} = A^{-1}\vec{b}$.
- If $A\vec{x} = \vec{0}$ for $\vec{x} \neq \vec{0}$, A cannot be invertible.
  That is, if A is invertible, $A\vec{x} = \vec{0}$ happens only when $\vec{x} = \vec{0}$

(Compare to def. of independent: A is invertible if its columns are independent).