Usually we are not so lucky to have Finite Fourier series, so this may be an approximation of the values of our function. (Like using a Taylor polynomial to approximate a Taylor series).

We can go in reverse. To reconstruct our function given some test values, use $F_n^{-1} = \frac{1}{n} F_n^H$.

$$\frac{1}{n} F_n^H \begin{bmatrix} z_0 \\ z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

recovers the coefficients of the Fourier series that takes values

$f(0) = z_0$
$f(\omega_4) = z_1$
$f(\omega_4^2) = z_2$
$f(\omega_4^3) = z_3$.

Next time: FFT (Fast Fourier transform) — not on HW, so don't worry.

<u>Def</u>: A is <u>normal</u> if $A^* A = AA^*$.

Fact: A is normal iff it can be diagonalized w/ orthonormal eigenvectors.

Class 25: 12/6

Final exam: 9-12 on 12/21 in Wilson 302
Office hours: 12-2 on 12/19 and 12/20 in Kasser 105.

<u>Recall</u>: Fourier series from last time.
(Fourier matrix)(coefficients) = (values of function)

Talk about reversing the process

$$F_n^{-1} = \frac{1}{n} F_n^H \quad \text{because} \quad 1 + \omega_n + \omega_n^2 + \dots + \omega_n^{n-1} = 0.$$

<u>Fast Fourier transform</u>:
Usually, multiplying $A\vec{x}$ takes n^2 multiplications and some additions. We want to do fewer by having lots of 0's in our matrices.

F_n has no zeros. But F_n = a product of sparse matrices.

Example:
$$F_4 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & i \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -i \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & i^2 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & i^2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \left[\begin{array}{c|c} I_2 & D_2 \\ \hline I_2 & D_2 \end{array}\right] \left[\begin{array}{c|c} F_2 & O \\ \hline O & F_2 \end{array}\right] \left[\text{Permutation}\right]$$

This cuts the multiplications in half