Class 19: 11/13
Midterm 2 TONIGHT!
Review day.

Class 20: 11/15
Announcements: Exam results

Outline: I. Markov matrices
II. Exam questions

I. Recall last time: problem about people switching political parties.
Party 1 always ended up doing better: 60% to 40%.

Start with facts about Markov Matrices
(prepared last time).
and examples

Definition: A Markov matrix is regular if A^k is positive for some $k \geq 1$ (i.e. A^k has all entries > 0).
In this case, there is a steady state.

Theorem: If A is a regular Markov Matrix, $A^\infty = \lim_{k\to\infty} A^k$ is the rank 1 matrix whose columns are all equal to the steady state vector.

(Why? $\lim_{k\to\infty} A^k \begin{bmatrix}1\\0\\0\\0\\\vdots\end{bmatrix}$ = steady state = $\lim_{k\to\infty} A^k \begin{bmatrix}0\\1\\0\\0\\\vdots\end{bmatrix} = \lim_{k\to\infty} A^k \begin{bmatrix}0\\0\\1\\0\\\vdots\end{bmatrix}$ etc)

So all columns of A^∞ are the steady state.

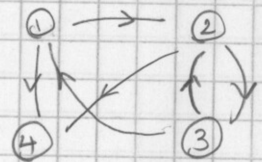
Application: Google page rank.
Idea: Click on links at random and see which websites get visited most.

- If you're on a website, choose one of the links (each link has equal probability) to get to the next website.
- If your website links to no other websites, choose another website at random and start clicking again.

We can construct a Markov matrix A so that
$A^k u_0$ = "where you end up after k clicks if you start at u_0"

(" " because it won't be one website; it will be the probabilities that you get to each site).

ample:



$$A = \begin{bmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{1}{2} & 0 & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{4} \end{bmatrix}$$

I assure you that A is regular. (A^3 is positive)
What is the steady state?

Compute $N(A-I)$: $$A - I = \begin{bmatrix} -1 & 0 & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & -1 & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{1}{2} & -1 & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} & 0 & -\frac{3}{4} \end{bmatrix}$$

Row reduce: $\vec{s}_1 = \begin{bmatrix} \frac{6}{10} \\ \frac{9}{10} \\ \frac{7}{10} \\ 1 \end{bmatrix}$ entries sum to $\frac{32}{10}$, not 1

Steady state: $\frac{10}{32}\vec{s}_1 = \begin{bmatrix} \frac{6}{32} \\ \frac{9}{32} \\ \frac{7}{32} \\ \frac{10}{32} \end{bmatrix}$ So 4 has the highest page rank
2 comes in a close second.

Note: For Markov Matrix problems, it is easy to do way more work than necessary.

- Do you need to find eigenvalues? (sometimes, not always)
- If you need to find eigenvalues, did you use that $\lambda_1 = 1$?
- Do you need to find all eigenvectors?
  (sometimes, just find the steady state, i.e. $\vec{x}_1$ corresponding to $\lambda_1 = 1$)