Math 520 - Practice Problems for Midterm I

For Full Credit, Show All Work
No Calculators

1. Let
$$A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 3 & 0 \\ 3 & 6 & 3 \end{bmatrix}.$$
Compute A^{-1}.

2. Find an orthonormal basis for the space spanned by the vectors
$$\begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 4 \\ 1 \end{bmatrix}$$
Let A be the matrix with these vectors as columns. Factor $A = QR$.

3. For what values of c does the following system have a solution?
$$\begin{bmatrix} 3 & 1 & 6 \\ 2 & 0 & 4 \\ 1 & 2 & 2 \end{bmatrix} \vec{x} = \begin{bmatrix} 1 \\ c \\ c \end{bmatrix}$$

4. What is the dimension of the space of 2×2 symmetric matrices? Give a basis for this vector space.

5. Let $\mathbf{v}_1 = (1, 2, 0, 1), \mathbf{v}_2 = (1, 1, 1, 1),\ \mathbf{v}_3 = (2, 3, 1, 0)$.
   (a) Find a vector in $\mathbb{R}^4$ that is perpendicular to $\mathbf{v}_1, \mathbf{v}_2$, and $\mathbf{v}_3$.
   (b) For which values of c is the 4×4 matrix whose columns are $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$, and $(1, -1, c, 0)$ invertible?

6. Let A be a 5×4 matrix.
   (a) Let $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ be vectors in $\mathbb{R}^4$. By the definition of linear independence, what does it mean for the vectors $A\mathbf{v}_1, A\mathbf{v}_2$, and $A\mathbf{v}_3$ in $\mathbb{R}^5$ to be linearly independent?
   (b) Suppose now that $N(A) = 0$ and that $\mathbf{v}_1, \mathbf{v}_2$, and $\mathbf{v}_3$ are linearly independent vectors in $\mathbb{R}^4$. Show that $A\mathbf{v}_1, A\mathbf{v}_2$,and $A\mathbf{v}_3$ are linearly independent vectors in $\mathbb{R}^5$.