

Assignment 6 (Due 10/18).

§5.1

4) $J_3 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ is the identity with rows 1 and 3 switched.
(one row switch, so $\det = -1$)

$J_4 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ is the identity with rows 1 and 4 switched
and rows 2 and 3 switched.
(two row switches, so $\det = +1$)

9) $A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ has determinant 1
(exchange rows 1 and 2, then rows 2 and 3 to get I).

$B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ has determinant 2.

Row reduce:

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & -2 \end{bmatrix}$$

Permute
(changes
sign of
det).

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -2 \end{bmatrix}$$

$$\text{So } \det = (-1) \cdot 1 \cdot 1 \cdot (-2) = 2.$$

$C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ has determinant 0 since it has rows that are equal.

14)

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 6 & 6 & 1 \\ -1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 7 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 2 & 3 & 3 \\ 0 & 2 & 0 & 7 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$

$$\det A = 1 \cdot 2 \cdot 3 \cdot 6 = 36$$

$$B = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & 3/2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & 3/2 & -1 & 0 \\ 0 & 0 & 4/3 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & 3/2 & -1 & 0 \\ 0 & 0 & 4/3 & -1 \\ 0 & 0 & 0 & 5/4 \end{bmatrix}$$

$$\det B = 2 \cdot \frac{3}{2} \cdot \frac{4}{3} \cdot \frac{5}{4} = 5$$

85.2

12)

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

$$AC^T = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} = 4I$$

$$C^T = 4A^{-1}$$

13)

$$a) \quad C_1 = |0| = 0$$

$$C_2 = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1$$

$$C_3 = \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = 0 + 0 + 0 - 0 - 0 - 0 = 0$$

$$C_4 = \begin{vmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix} = -1 \cdot \begin{vmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = -1(-1) = 1.$$

$$b) \quad C_n = \begin{vmatrix} \overbrace{0 & 1 & 0 & \dots & 0}^n \\ \overbrace{1 & 0 & 1 & 0 & \dots & 0}^{n-1} \\ 0 & 1 & 0 & 1 & \dots & 0 \\ 0 & 0 & 1 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 & 0 \end{vmatrix} = -1 \begin{vmatrix} \overbrace{1 & 1 & 0 & \dots & 0}^{n-1} \\ 0 & 0 & 1 & 0 & \dots & 0 \\ 0 & 1 & 0 & 1 & \dots & 0 \\ 0 & 0 & 1 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 & 0 \end{vmatrix} \Bigg\}^{n-1}$$

$$= -1 \cdot \left(1 \cdot \begin{vmatrix} \overbrace{0 & 1 & 0 & \dots & 0}^{n-2} \\ \overbrace{1 & 0 & 1 & \dots & 0}^{n-3} \\ 0 & 1 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 1 & 0 \end{vmatrix} \Bigg\}^{n-2} - 1 \cdot \begin{vmatrix} \overbrace{0 & 1 & 0 & \dots & 0}^{n-2} \\ \overbrace{0 & 0 & 1 & \dots & 0}^{n-3} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 1 & 0 \end{vmatrix} \right)$$

$$= -1 \cdot C_{n-2} - 0$$

$$= -C_{n-2}$$

$$\text{So } C_{10} = -C_8 = C_6 = -C_4 = -1$$