1. Evaluate $\int \frac{x-8}{x^3+4x}\,dx$.

$$\frac{x-8}{x^3+4x} = \frac{x-8}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$$

$$A(x^2+4) + (Bx+C)x = x-8$$

Set $x=0$:

$$4A = -8 \longrightarrow A = -2.$$

So $Bx^2 + Cx = x - \cancel{8} + 2x^2 + \cancel{8}$

$$B = 2, \quad C = 1.$$

$$\int \frac{x-8}{x^3+4x}\,dx = \int \frac{-2}{x} + \frac{2x}{x^2+4} + \frac{1}{x^2+4}\,dx$$

$$= -2\ln|x| + \ln|x^2+4| + \int \frac{1}{x^2+4}\,dx$$

$$= -2\ln|x| + \ln|x^2+4| + \frac{1}{4}\int \frac{1}{\left(\frac{x}{2}\right)^2+1}\,dx$$

$$u = \frac{x}{2} \quad du = \frac{dx}{2}$$

$$= -2\ln|x| + \ln(x^2+4) + \frac{1}{4}\int \frac{2}{u^2+1}\,du$$

$$= \boxed{-2\ln|x| + \ln(x^2+4) + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C}$$

2. Approximate the integral $\int_{-2}^{2}(x^3 - 2x + 1)dx$ using Simpon's rule with 4 subintervals.

$$S = \frac{\Delta x}{3}\left(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)\right)$$

$$= \frac{1}{3}\left(f(-2) + 4f(-1) + 2f(0) + 4f(1) + f(2)\right)$$

$$= \frac{1}{3}\left(-3 + 4(2) + 2(1) + 4(0) + 5\right)$$

$$= \frac{1}{3}(12) = \boxed{4}$$

3. Evaluate $\displaystyle\int \frac{1}{(25-x^2)^{3/2}}\,dx$.

Set $x = 5\sin\theta \qquad \left(-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}\right)$.

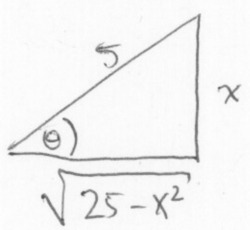
$dx = 5\cos\theta\, d\theta$

$$\int \frac{1}{(25-x^2)^{3/2}}\,dx = \int \frac{5\cos\theta}{(25\cos^2\theta)^{3/2}}\,d\theta$$

$$= \int \frac{5\cos\theta}{5^3|\cos\theta|^3}\,d\theta$$

$$= \frac{1}{25}\int \frac{1}{\cos^2\theta}\,d\theta = \frac{1}{25}\int \sec^2\theta\, d\theta$$

$$= \frac{1}{25}\tan\theta + C.$$



$$= \boxed{\frac{x}{25\sqrt{25-x^2}} + C}$$

4. Evaluate $\int e^x \sin(2x)\,dx$.

Use integration by parts:

$$u = e^x \qquad du = e^x\,dx$$
$$dv = \sin(2x)\,dx \qquad v = \frac{-1}{2}\cos(2x)$$

$$\int e^x \sin(2x)\,dx = \frac{-e^x}{2}\cdot\cos(2x) + \frac{1}{2}\int e^x \cos(2x)\,dx.$$

$$\int e^x \cos(2x)\,dx = \frac{1}{2}e^x \sin(2x) - \frac{1}{2}\int e^x \sin(2x)\,dx.$$

$$u = e^x \qquad du = e^x\,dx$$
$$dv = \cos 2x\,dx \qquad v = \frac{1}{2}\sin(2x)$$

So $$\int e^x \sin(2x)\,dx = \frac{-e^x\cos(2x)}{2} + \frac{1}{4}e^x\sin(2x) - \frac{1}{4}\int e^x \sin 2x\,dx$$

$$\frac{5}{4}\int e^x \sin(2x)\,dx = \frac{e^x\sin(2x) - 2e^x\cos(2x)}{4}$$

$$\int e^x \sin 2x\,dx = \boxed{\frac{e^x\sin(2x) - 2e^x\cos(2x)}{5} + C}.$$

5. Evaluate $\int_0^1 \frac{e^{3x}}{25e^{6x}+e^{3x}}\,dx$ using the formula $\int \frac{1}{u(au+b)}\,du = \frac{1}{b}\ln\left|\frac{u}{au+b}\right| + c.$

$u = e^{3x}$ upper bound: $u = e^3$

$du = 3e^{3x}\,dx$ lower bound: $u = e^0 = 1.$

$$\int_0^1 \frac{e^{3x}}{25e^{6x}+e^{3x}}\,dx = \frac{1}{3}\int_1^{e^3} \frac{1}{u(25u+1)}\,du$$

$$= \frac{1}{3}\ln\left|\frac{u}{25u+1}\right|\Bigg|_1^{e^3}$$

$$= \frac{1}{3}\left(\ln\left|\frac{e^3}{25e^3+1}\right| - \ln\left|\frac{1}{26}\right|\right)$$

$$= \boxed{\frac{1}{3}\ln\left(\frac{26e^3}{25e^3+1}\right)}$$

6. Find the values of p for which the following integral converges and evaluate it for those values of p:

$$\int_0^1 x^p \ln x \, dx.$$

$$\int_0^1 x^p \ln x \, dx = \lim_{t \to 0^+} \int_t^1 x^p \ln x \, dx$$

If $p = -1$:

$$\lim_{t \to 0^+} \int_t^1 \frac{\ln x}{x} dx = \lim_{t \to 0^+} \int_{x=t}^{x=1} u \, du = \lim_{t \to 0^+} \left(\frac{(\ln x)^2}{2}\right)\Big|_t^1$$

$u = \ln x$
$du = \frac{1}{x} dx$

$$= \lim_{t \to 0^+} -\frac{(\ln t)^2}{2} = \infty.$$

So if $p = -1$, the integral diverges.

If $p < -1$, then $x^p \ln x \geq \frac{\ln x}{x}$ on $[0,1]$.

So if $p < -1$, $\int_0^1 x^p \ln x \, dx$ diverges by the direct comparison test.

If $p > -1$:

$$\int_0^1 x^p \ln x \, dx = \lim_{t \to 0^+} \int_t^1 x^p \ln x \, dx$$

$u = \ln x \qquad du = \frac{1}{x} dx$
$dv = x^p dx \qquad v = \frac{1}{p+1} x^{p+1}$

$$= \lim_{t \to 0^+} \frac{(\ln x) x^{p+1}}{p+1}\Big|_t^1 - \int_t^1 \frac{x^p}{p+1} dx =$$

$$= \lim_{t \to 0^+} \left(0 - \frac{\ln(t) \cdot t^{p+1}}{p+1}\right) - \frac{x^{p+1}}{(p+1)^2}\Big|_t^1$$

$$= \lim_{t \to 0^+} -\frac{\ln(t) \cdot t^{p+1}}{p+1} - \frac{1}{(p+1)^2} + \cancel{t^{p+1}}^{0}$$

$$= \lim_{t \to 0^+} -\frac{\ln(t)}{(p+1)\, t^{-(p+1)}} - \frac{1}{(p+1)^2} = \lim_{t \to 0^+} -\frac{\frac{1}{t}}{-(p+1)^2 t^{-p-2}} - 1$$

$$= \lim_{t \to 0^+} \frac{t^{p+1}}{(p+1)^2} - \frac{1}{(p+1)^2} = \frac{-1}{(p+1)^2}.$$ So the integral converges.

6. continued.

So the integral converges if $p > -1$, and the integral is $\boxed{\frac{-1}{(p+1)^2}}$ in that case.

The integral diverges if $p \leq -1$.