Teaching Portfolio

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I. Statement of Teaching Philosophy

In the first class for which I was a TA, Calculus I, I gave a quiz asking students to find the volume of a solid in two ways, once integrating with washers and once integrating with cylindrical shells. When they turned in the quizzes, I said cheerfully that they should be confident this week because they would have gotten the same answer twice. There was an audible gasp in the room, and I realized that my students' math teachers, including myself, had failed them. Although they might never need to compute volumes using the method of cylindrical shells in their careers as doctors, engineers and bankers, they would be expected to be able to recognize their mistakes by thinking critically about problems. My students had been trained to thoughtlessly solve problems and rush to answers without actually considering what they were doing and why. My experience as a TA for that class made me realize what I really want my students to get out of their math classes: *the critical thinking skills necessary to recognize and correct mistakes, and think through and solve new and challenging problems.*

Since being a TA for that Calculus I class, I have lectured for a wide range of classes: from Calculus with Analytic Geometry, the class most likely to be taken by non-physical sciences majors, to Introduction to Number Theory, a course for non-majors. In addition, I volunteer in an Algebra I classroom at Hope High School, a Providence public school. In all my teaching experiences, I have worked to impart these critical thinking skills. To foster deeper understanding of the material in the Calculus with Analytic Geometry class, I included some word problems on every homework assignment and exam, forcing students to be able to apply the concepts we were learning to the real world. For instance, they applied the derivative to problems involving velocity and modeled population growth with differential equations. Although they struggled with word problems and applications at first, it got easier with practice. In the end, in departmental evaluations, one third of respondents mentioned problem solving or conceptual understanding in response to the question "What knowledge or skills did this course help you develop?"

For a more advanced class like Introduction to Number Theory, a large part of my goal was to involve students in the process of mathematical discovery. In particular, I wanted them to be faced with unfamiliar problems but learn strategies to start to think about and solve them. To accomplish this, I emphasized the importance of numerical experimentation, encouraging students to look for patterns and make conjectures. It was especially satisfying to see students apply this method of discovery on their own when faced with unfamiliar problems. In one instance, a student noticed a pattern that I didn't know previously, but which Gauss described and proved in his *Disquisitiones Arithmeticae*. Proving this student's conjecture became an extra credit assignment which a handful of students completed. I got many positive comments on evaluations indicating that students felt they had developed problem solving and proof writing skills in this course, including one student's response: "This course helped me develop analytical skills that I had not previously needed to complete my Calculus courses." Over the course of this class, I saw my students become inquisitive and develop the tools to participate in the process of mathematical discovery and proof.

Developing critical thinking skills is not easy, so I make every effort to be responsive to my students' needs. For the last two classes I taught, I asked students to write me a letter on the first day of class explaining what they wanted to get out of it and their plan for succeeding in the course. In the Calculus with Analytic Geometry course, I found that many of my students had struggled with math, but wanted to be able to apply calculus to their daily lives and to courses in their majors. In response to that, I made sure to emphasize ways in which students could get help,

reminding them about office hours and being as flexible as possible about meeting students outside of office hours. In a calculus course, when students told me that they wanted more opportunities for assessment, I started making weekly ungraded quizzes that they could use to make sure they were keeping up with the material. Of my caring about my class, a student from my number theory class wrote: "She had weekly office hours, but she never cut them short if students had more questions or concerns; I can recall several occasions in which we went an hour or two over the scheduled time... She really wanted all of us to succeed." In general, I find that students respect and appreciate the work I put into the class and respond by stepping up and working hard themselves. Because I want my students to become good problem solvers, which is no easy task, their hard work is crucial.

In my ideal classroom, students take ownership of the material and in doing so, learn that math is something they can both do and create. When I taught number theory, I had a problem due every class period called the Problem of the Day. These problems were chosen to encourage exploration and conjectures, such as one that asked students to explore which integers less than 100 could be written as the sum of two squares, and whether the product of two sums of squares was again a sum of squares. At least half of each class was driven by student observations and ideas from these problems. Even in the half of class that was mostly lecturing, I asked for input on topics such as methods of proof, making it the norm that students would suggest ideas and ask questions. For my Calculus with Analytic Geometry class, every Friday was set aside for student questions as a "recitation day." If we finished answering all the questions, which I usually tried to get other students to answer, I had a worksheet prepared for students to solve in groups and discuss as a class. I believe that when students take ownership of the material, they become much more motivated to delve into topics, learning them at the deep level which is required for the analytical skills I hope they will develop.

II. Teaching Activities

Semester-long Courses

These are courses for which I ran lecture sessions, either as one of many lecture sections, or independently.

Fall 2012: Linear Algebra (Math 0520)

I prepared and gave lectures to my section of about 27 students (mine was one of three coordinating sections). I contributed to choosing homework assignments and writing exams. In addition, I coordinated and proctored exams for students who needed testing accommodations.

Official Course Description: Vector spaces, linear transformations, matrices, systems of linear equations, bases, projections, rotations, determinants, and inner products. Applications may include differential equations, difference equations, least squares approximations, and models in economics and in biological and physical sciences.

Textbook: *Introduction to Linear Algebra*, 4th ed., by Gilbert Strang

Spring 2012: Analytic Geometry with Calculus (Math 0060)

I wrote the syllabus and designed the course, preparing lectures and setting weekly homework assignments. In addition, I wrote weekly quizzes, designed worksheets and in-class activities, and held office hours.

Official Course Description: A slower-paced introduction to calculus for students who require additional preparation for calculus. This sequence presents the same calculus topics as [Introductory Calculus I], together with all the necessary pre-calculus topics. Students successfully completing this sequence will be prepared for [Introductory Calculus II].

Textbook: *Calculus I with Precalculus*, 3rd ed., by Ron Larson et al.

Spring 2011: Introduction to Number Theory (Math 0420)

I wrote the syllabus and designed the course, preparing lectures and writing weekly problem sets. In addition, I wrote "problems of the day," one or two problems due every class period which were designed to provoke thought, inspire conjectures, and drive class discussion.

Official Course Description: This course will provide an overview of one of the most beautiful areas of mathematics. It is ideal for any student who wants a taste of mathematics outside of, or in addition to, the calculus sequence. Topics to be covered include: prime numbers, congruences, quadratic reciprocity, sums of squares, Diophantine equations, and as time permits, such topics as cryptography and continued fractions. No prerequisites.

Textbook: None (by choice)

Fall 2010: Advanced Placement Calculus (Math 0170)

I prepared and gave lectures to my section of 22 students (mine was one of 5 coordinated lecture sections). In addition, I contributed to writing exams and wrote weekly ungraded quizzes for my section.

Official Course Description: This course begins with a review of fundamentals of calculus, and includes infinite series, power series, paths, and differential equations of first and second order. Placement in this course is determined by the department on the basis of high school AP examinations scores or the results of tests given by the department during orientation week.

Textbook: *University Calculus, Single Variable with Vectors* by J. Hass, M. Weir, G. Thomas

Teaching Assistantships

These are courses for which I was a TA and ran recitation sections.

Spring 2010: Introductory Calculus II (Math 0100)

I ran two weekly recitation sections, each with about 20 students. I wrote and graded weekly quizzes.

Official Course Description: An intensive course in the calculus of one variable including limits; differentiation; maxima and minima, and the chain rule for polynomials, rational functions, trigonometric functions, and exponential functions. Introduction of integration with applications to area and volumes of revolution.

Textbook: *Calculus and Early Transcendentals*, 6th ed., by James Stewart

Fall 2009: Introductory Calculus I (Math 0090)

I ran two weekly recitation sections, each with about 30 students. I wrote and graded weekly quizzes. In addition, I coordinated and proctored exams for students with testing accommodations.

Official Course Description: A continuation of the material of [Introductory Calculus I] including further development of integration, inverse trigonometric and logarithmic functions, techniques of integrations, and applications which include work and pressure. Other topics covered are infinite series, power series, Taylor's formula, polar coordinates, parametric equations, introduction to differential equations, and numerical methods.

Textbook: *Calculus and Early Transcendentals*, 6th ed., by James Stewart

Summer Activities

Summer 2010: Summer@Brown

With Jackie Anderson, I designed and ran a 3-week summer course, Number Theory: Introduction to Higher Mathematics, for high school students in the Summer@Brown program. We wrote problem sets, designed in-class activities and shared lecturing and grading responsibilities. We had 26 students.

Summer 2010: PROMYS for Teachers

As an undergraduate, I was a counselor in the PROMYS program (a 6-week summer number theory program for gifted high school students held at Boston University). PROMYS also has a program for secondary school teachers, and I was a counselor in the summer of 2010. I graded my students' daily problem sets and worked with them on the process of mathematical discovery.

Service and Other Activities

Volunteer in Hope High School

Since Fall 2011, I have volunteered at Hope High School, a public high school in Providence, with the Brown service group Algebra in Motion (AiM). During Fall 2011, I participated in the part of AiM that provides free after-school drop-in math tutoring. I went once a week for 2-3 hours at a time and helped students with homework in Algebra I, Geometry and Precalculus.

In Spring 2012, I volunteered in an Algebra I classroom, going to Hope High twice a week and acting as a teacher's aide. I worked with struggling students during class. I continue to be an in-class tutor during Fall 2012, but now I'm assisting in a Math Models class. Math Models is a course to prepare students for Algebra II and is aimed at students who did poorly in Algebra I and Geometry.

Sheridan Center for Teaching and Learning

In an effort to improve my teaching, I have been involved with the Sheridan Center in various ways since Fall 2009, and I have been the graduate student liaison to the Sheridan Center since Summer 2011. In particular, I have done the following.

- **Certificate Programs**: I completed Certificate I (Reflective Teaching) during the 2009-2010 academic year and Certificate IV (Teaching Consultant Program) during the 2011-2012 academic year. I am currently enrolled in Certificate III (Professional Development Seminar).
- **Teaching Consultant**: As a teaching consultant for the Sheridan Center, I have observed and provided feedback for numerous lectures, presentations, and practice teaching sessions across the Brown campus. At the start of this semester, I was one of a number of teaching consultants who ran interactive workshops on Teaching for the First Time for the Sheridan Center's New TA Orientation. I have also attended special workshops and lectures on reflective learning and other topics.
- **Graduate Student Liaison**: My duties as graduate student liaison to the Sheridan Center include helping to run departmental new TA training and making other graduate students aware of Sheridan Center programs and events.

Expository Talks for Undergraduates

I have given two expository talks for undergraduates

- Where Geometry and Number Theory Collide!, given at Bowdoin College, March 30, 2010. This talk introduced Minkowski's theorem and the geometry of numbers.
- Bernoulli Numbers and their Applications, given to the Undergraduate Math Club at the University of Connecticut, November 18, 2009. This talk introduced Bernoulli numbers and described their relationship to special values of the Riemann zeta function.

III. Evidence of Teaching Effectiveness

Selected Comments from Student Evaluations

These comments are taken from end-of-semester evaluations of my Linear Algebra, Analytic Geometry with Calculus, Introduction to Number Theory, and Advanced Placement Calculus courses taught at Brown. They show that my students gained problem solving skills, how I work hard to help my students, and that I am passionate about the material I teach. I find it very encouraging that students feel that they have gained critical thinking skills and that they appreciate my dedication because it shows that I am on track with accomplishing the goals I discuss in my teaching philosophy statement.

About Skills Acquired

- (I gained) lots of linear algebra knowledge! Also how to take good math notes.
- I felt like I learned a lot about the material and have a strong grasp of the concepts.
- The course was very effective in the sense that it allowed me to developed [sic] the problem solving skills that I wanted to focus on.
- The course helped me increase my confidence in my math skills. It taught me that calculus is do-able for me.
- Professor Li-Mei Lim was the first teacher to give me a complete understanding of calculus and analytic geometry. I now finally can grasp all the aspects of trigonometry, integrals, anti/derivatives, and fundamental calculus concepts and I can understand how important they are to my every day life. I feel very prepared to move on to higher mathematic [sic] courses and I am excited for the challenges ahead.
- I learned to develop several different problem solving skills I did not acquire from math courses I took in high school.
- This course helped me develop analytical skills that I had not previously needed to complete my Calculus courses. I learned various techniques for solving proofs such as induction and contradiction.

About My Caring for Students

- Li-Mei is an incredible instructor! She is so dedicated to helping out each and every student. She always went out of her way to make sure every question was answered. She held very useful office hours. Her clarity and shear knowledge was unbelievably good!
- Li-Mei was very effective and really cared about the class. Did anything to help a student who needed it.
- Professor Li-Mei Lim was always open to questions and suggestions. She would send out notices for us to give her suggestions on what everyone was having trouble with so she could integrate those concepts into her teaching plan.
- Li-Mei may be Gods gift to math students... I think what really makes her different is she cares about each student. By the first week of class Li-Mei knew everyones [sic] name. She was incredibly self aware of the class a trait other teachers truly lack. If she realized we weren't grasping a concept she would stop and try to teach it another way. Her encouragement for class participation was unrivaled.
- Instructor was clear, willing to answer questions, and enthusiastic. She effectively engaged the class and was invested in our understanding of the material.
- Li-Mei was an incredible instructor. She was not only engaging and articulate during lecture, but she was furthermore always encouraging her students to delve further, ask questions, and offer ideas. She had weekly office hours, but she never cut them short if students had more questions or concerns; I can recall several occasions in which we went an hour or two over the scheduled time for office hours... She really wanted all of us to succeed.

About My Enthusiasm

- She was very excited about linear algebra, and it was contagious.
- Li-Mei Lim is a great instructor. She is excited about the course and transfers that to her students.
- Li-Mei is an amazing professor, one that I would wholeheartedly recommend. She is passionate about the subject matter and wants everyone to learn, enjoy, and understand it.
- She was very enthusiastic and willing to help clarify points. Unlike the other teachers of the course, she made practice exams, quizzes and a final.
- I liked how she seemed to enjoy the material and wanted everyone else to see that it was fun too.

Aggregate Statistics

These statistics are taken from the end-of-semester evaluations. The scales goes from 1 (excellent) to 5 (very poor). Courses indicated with an asterisk (*) are courses I designed and ran independently. Note that the courses I had complete control over were more highly rated than the course (Linear Algebra) for which I did not control the syllabus and course structure.

Course	Effectiveness of Course	Effectiveness of Instructor
Linear Algebra	1.88	1.44
Analytic Geometry with Calculus*	1.73	1.45
Number Theory*	1.70	1.43

Brown Critical Review

The following is an excerpt from the review of my Spring 2011 course, Introduction to Number Theory, from the Brown Critical Review. The Critical Review is a student-run organization that publishes course review based on student surveys for the Brown community.

> "Introduction to Number Theory" was taught by Professor Lim, whom students described as a very engaging lecturer. A number of students found Lim's combination of proofs, examples, assignments, and lecture to be very effective in teaching the material. Respondents commented that Professor Lim "encouraged questions constantly and successfully," and explained topics as much as necessary. The professor was described as very available outside of class and students commented that she always gave good feedback on assignments.
>
> The course had ten homework assignments, two midterms, a final exam, and a daily "problem of the day." Students found the assignments pertinent and helpful for understanding the material, especially since there was no textbook for the course.
>
> Students generally spent an average of four hours a week on the course, and up to five to seven hours on some weeks. Most class members could find no fault with the course, though some suggested the addition of readings to help further understanding outside of lecture. Almost all of the students encouraged any prospective student to take this fantastic course!

On a scale from 1 to 4, 1 being positive/agree and 4 being negative/disagree, the course was rated 1.27 and the instructor was rated 1.19. The rating for "learned a lot" was 1.32.

Unsolicited Feedback

After submitting grades, I got the following email from a student in my Linear Algebra class. We had met several times to discuss his progress in the class, and he came to office hours on a semi-regular basis. He received a C in the class, the lowest passing grade possible at Brown.

> Congratulations! We did it, and I did not fail Math52. It may have been a stressful struggle throughout the semester, but I'm glad to have gone through the course with you, and could not have imagined doing it with anyone else. I don't want you to judge your teaching abilities by my grades in the course, because I really feel I learned everything I came into the course to learn, and you really have been the best.

This is the syllabus for the Calculus with Analytic Geometry course I taught at Brown. This course is the second semester of a two-semester sequence that integrates precalculus with first-semester calculus. A main goal of the course was to equip students with problem solving skills so that they could apply calculus to their other courses and successfully move on to second-semester calculus. To that end, I instituted Friday "recitation days" so that students could ask questions and practice solving problems together in groups.

Math 6: Calculus with Analytic Geometry
Course Syllabus: Spring 2012

Instructor: Li-Mei Lim **Office:** KH 018
Email: llim@math.brown.edu **Office Hours:** TBD
Website: `math.brown.edu/~llim`

Course Details
The course will meet MWF 9am in BH 157. Generally, homework will be due on Wednesdays and quizzes will be on Fridays. In addition, Fridays will be "recitation days," where we will answer questions from the week and do extra practice exercises.

Topics to be Covered
We will use the same textbook as Math 5, *Calculus I with Precalculus*, third edition by Ron Larson et al., and cover chapters 6, 8, 9, 10, and 11.

Requirements
- **Homework:** Homework will be given weekly and will generally be due on Wednesdays. Odd-numbered problems, whose answers can be found in the back of the book, will be graded for completeness only and will be worth 2 points each. You should verify your own answers to check comprehension before going on to the even-numbered problems. Even-numbered problems will be graded for correctness, and will be worth 4 points each. You must show your work to receive full credit, on both odd and even numbered problems. If you work with classmates, please list the names of your collaborators on your assignment. ***No late assignments will be accepted.***
- **Quizzes:** There will be weekly quizzes, generally given on Fridays in class. ***No make-up quizzes will be given.***
- **Exams:** There will be three exams: two in-class midterms and a final. Midterms are tentatively scheduled for February 22 and March 21. The final exam is on May 14, 2012 at 2pm.
- **Attendance:** While attendance is not technically required, you are strongly encouraged to attend all classes. You are responsible for getting notes from a classmate if you miss class.

Grading
This class is Pass/No Credit for all students. Final grades will be determined using the following breakdown:

- Homework: 20%
- Quizzes: 15%
- Midterm 1: 20%
- Midterm 2: 20%
- Final Exam: 25%

Getting Help
To get the most out of this class, it is important that you ask questions when you have them and get all the help you need. Here is a list of suggestions and resources.

- **Ask questions during class.** You are strongly encouraged to ask questions during class. Don't be afraid to look stupid; chances are someone else has the same question.
- **Come to office hours.** Office hours are great for getting more one-on-one help. Come ask questions about homework, past quizzes and exams, or topics from class.
- **Read the textbook.** The textbook is filled with examples. Maybe one will answer your question. When reading, try taking notes, highlighting, and writing down questions to ask in class or in office hours to make the most of your time.
- **Go to MRC.** The Math Resource Center (MRC) is held in Kassar 105 Monday through Thursday 8-10pm. It is staffed by grad students and advanced undergrads who can help you with your homework or clarify points from class. Be aware that some nights can be very busy, so start your homework early. See **math.brown.edu/mrc** for more information.
- **Form study groups.** Your peers are a wonderful resource. Explaining your work to others will help you to learn the material better, while helping your classmates too. It's win-win! Just be sure to write solutions up on your own.

Academic Honesty

Collaboration on problem sets is encouraged, but solutions must be written up individually you must list the people you worked with. Books, notes, calculators, and other aids will not be allowed on any quizzes or exams. You are expected to follow Brown's Academic and Student Conduct Codes. Any violation of Brown's codes or class policies will result in automatic failure in this class and may result in further action.

This is an example of an activity I designed for my Calculus with Analytic Geometry class. I made quadrants with card stock, straws, and string weighted with pennies. I chose to do this activity because my students had expressed that it was important to them to be able to apply what they learned in a broader context. I also saw this activity as an opportunity to show students who had never liked math that math can be fun and is not just something one learns passively from lectures and books.

Math 6 – Trigonometry Outdoor Activity
14 March 2012

Today we'll measure the heights of some things using trigonometry.

First, measure the angle to the sun using your quadrant. **Don't look at the sun.** Instead, point the straw in the direction of the sun, and adjust the angle until the straw casts no shadow. If you hold a piece of paper a few inches from the end of the straw, you should see a circle of light at this point. Record the angle you measured here: _____

First we'll measure the height of something whose height we know. This way, we can see how accurate our measurements are.

Measure the length of your shadow (or the shadow of someone in your group). Record the length of the shadow here: _____

Draw a diagram relating the length of the shadow, your height, and the angle to the sun.

Use trigonometry to estimate your height.

How close is this to your actual height?

Now go measure other shadows and find the heights of other objects! Can you find the height of Barus and Holley? The tallest nearby tree? The SciLi?

This is the course syllabus for the Introduction to Number Theory course I taught at Brown. Because it was an elective course for non-majors, I wanted to show students what "real math" is like and engage them in the process of mathematical discovery. To that end, I gave a "problem of the day" each class which was meant to provoke thought and conjectures. The students' observations and conjectures drove every class.

Math 42: Introduction to Number Theory
Course Syllabus: Spring 2011

Instructor: Li-Mei Lim
Email: llim@math.brown.edu
Website: `www.math.brown.edu/~llim`
Office: Kassar 018
Office Hours: TBD

Overview

Mathematics is the queen of the sciences, and number theory is the queen of mathematics.

-Carl Friedrich Gauss

One of the goals of this class is to understand this quotation–what is it about number theory that Gauss found so beautiful and intriguing? More broadly, we will see mathematics as an empirical science, building problem solving and reasoning skills.

There are no prerequisites for the course other than high school algebra and a healthy amount of intellectual curiosity. In fact, this class is geared toward non-majors with the goal of giving students who are not math majors a window into what beautiful math exists beyond what most people experience.

Topics to be Covered

- Modular Arithmetic
- Linear Diophantine Equations
- Prime Factorization in $\mathbb{Z}$ and $\mathbb{Z}[i]$
- Continued Fractions
- Divisibility
- Chinese Remainder Theorem
- Quadratic Reciprocity

Textbook

There is no textbook for the class. This is by design–one of the goals of the course is to show you that *you* can create mathematics for yourself and do not need a textbook to tell you the answers. To that end, this course will be driven by *you* and your observations without the need for a textbook. As the semester progresses, some references may be suggested as needed.

Requirements

- **Homework:** Homework consists of two parts: graded problem sets and ungraded problems. Graded problem sets will be given weekly. Ungraded problems will be given daily, with at most two due on a given day. While you will not be graded on the content of your solutions of daily problems, it is important that you do them, as they will be the jumping-off point for class discussion. ***No late assignments will be accepted.***
- **Exams:** There will be three exams: two in-class midterms (tentatively March 1 and April 12) and a final (on May 13 at 2pm, location TBA).
- **Attendance:** While attendance is not technically required, you are strongly encouraged to attend all classes because of the student-driven nature of the course. If for some reason you must miss class, please email ahead of time to arrange turning in the problem of the day. You are responsible for getting notes from a classmate.

Grading

- Homework: 20%
- Midterm 1: 25%
- Midterm 2: 25%
- Final Exam: 30%

There will be some opportunities for extra credit throughout the semester. You can receive extra credit for presenting ideas on problems of the day or doing extra credit problems on problem sets.

Course Schedule

This schedule may change as the semester progresses.

1/27: Introduction, What is Number Theory?
2/1: Linear Diophantine Equations, Euclidean Algorithm
2/3: Linear Diophantine Equations continued, Magic Box
2/8: Fundamental Theorem of Arithmetic, Unique Prime Factorization *(Pset 1 due)*
2/10: Modular Arithmetic, Linear Congruences, Units
2/15: Euler φ function, Powers, Order *(Pset 2 due)*
2/17: Fermat's Little Theorem, Euler's Theorem
2/22: **President's Day: No class**
2/24: Cryptography *(Pset 3 due)*
3/1: **Midterm 1**
3/3: Systems of congruences
3/8: Chinese Remainder Theorem *(Pset 4 due)*
3/10: Primitive roots, Orders revisited
3/15: Using tables of indices *(Pset 5 due)*
3/17: The Legendre symbol
3/22: Squares mod p: -1 *(Pset 6 due)*
3/24: $\mathbb{Z}[i]$: Norms, primes, units
3/29: **Spring Break-No class**
3/31: **Spring Break-No class**
4/5: $\mathbb{Z}[i]$ continued, Sums of squares *(Pset 7 due)*
4/7: $\mathbb{Z}[i]$, $\left(\frac{-1}{p}\right)$, Sums of squares
4/12: **Midterm 2**
4/14: $\left(\frac{2}{p}\right)$
4/19: Quadratic Reciprocity *(Pset 8 due)*
4/21: UPF revisited: $\mathbb{Z}[\sqrt{-5}]$, $\mathbb{Z}[\sqrt{2}]$
4/26: Continued Fractions of square roots *(Pset 9 due)*
4/28: Units in $\mathbb{Z}[\sqrt{D}]$, Super Magic Box
5/3: UPF revisited conclusion *(Pset 10 due)*
5/5: Catch-up day and/or review
5/10: Review

What to Do When You Don't Know What to Do

The process by which "real math" is created goes something like this:

1. Do some calculations. Try a few simple examples, then a few slightly more complicated examples.
2. Notice a pattern and write a precise statement (a conjecture) describing the pattern.
3. Test the conjecture by doing a few more examples.

4. Prove the conjecture (making it a theorem).

In this class, we will focus on steps 1 through 3 (repeating steps 2 and 3 as necessary until our conjectures are correct). While we won't prove everything, we will see some significant proofs.

If you are stuck on a problem, there are a few thing you could try.

- ***Go back to step 1***. Do a simpler example, and you may get some ideas. The value of experimentation cannot be stressed enough.
- Think about whether you have seen a similar problem before. Can old problems be applied in new ways?
- Form study groups. Bounce ideas off each other and see what sticks.
- Come to office hours. Everyone needs a little push in the right direction sometimes.

This is an example of a mini problem set given in my Number Theory course. A "problem of the day" like this one was given each class, and students' observations and conjectures would drive class the next day, getting them to engage in the process of mathematical discovery. These problems were also meant to show students that math is something they can create and do, not just something they absorb passively.

Math 42-Number Theory
Problem of the Day #4
Due Thursday, February 10, 2011

Consider for a moment what $\frac{1}{2}$ means. It is a number that satisfies $2x = 1$. In other words, $\frac{1}{2}$ is a number that when multiplied by 2 gives 1–it is the multiplicative inverse of 2. We can think about this idea in $\mathbb{Z}_m$. (Note also that $\sqrt{-1}$ means a number that satisfies $x^2 = -1$, and we can also think about this equation in $\mathbb{Z}_m$.)

1. Find $\frac{1}{2}$ in $\mathbb{Z}_7$. Find $\frac{2}{7}$ in $\mathbb{Z}_{11}$. Does $\sqrt{-1}$ exist in $\mathbb{Z}_5$, $\mathbb{Z}_7$, $\mathbb{Z}_{11}$ and $\mathbb{Z}_{13}$?

2. List all incongruent solutions mod 20 to the equation $6x \equiv 8 \mod 20$. (By "incongruent solutions mod 20" we mean find the solutions in $\mathbb{Z}_{20}$, that is, your solutions will all be between 0 and 19 inclusive.)