Answers

1) I’ve given just the solutions, without the logic spelled out.
   a) $^{52}\text{Fe} \rightarrow ^{52}\text{Mn} + _{\beta}\text{+ }\gamma$
   b) $^{123}\text{Te} + _{\beta}\text{→} ^{123}\text{Sb}$
   c) $^{59}\text{Fe} \rightarrow ^{59}\text{Co} + _{\beta}\text{+ }\gamma$
   d) $^{212}\text{Bi} \rightarrow ^{212}\text{Po} + _{\beta}$
   e) $^{8}\text{B} \rightarrow ^{8}\text{Be} + _{\beta}$

2) We can assume that all the Pb came from the U. Because of the mass difference, we need to convert mass into moles. We ended up with $\frac{0.00537\text{g}}{238\text{g/mol}} = 2.256 \times 10^{-5}$ moles $^{238}\text{U}$ and $\frac{0.00252\text{g}}{206\text{g/mol}} = 1.223 \times 10^{-5}$ mole $^{206}\text{Pb}$. Therefore, the total amount of $^{238}\text{U}$ that was present initially is $(2.256 + 1.223) \times 10^{-5}$ moles = $3.479 \times 10^{-5}$ moles. We can use these values for $N_t$ and $N_0$ in our equation $\ln \left[ \frac{N_t}{N_0} \right] = -kt$. Given the $t_{1/2} = 4.5 \times 10^9$ years, we can get the decay constant, $k = \frac{\ln(2)}{4.5 \times 10^9 \text{ yrs}} = 1.540 \times 10^{10} \text{ yr}^{-1}$. Now we can substitute what we know to get $t = \text{age of the ore} = \frac{\ln \left( \frac{2.256}{3.479} \right)}{-1.540 \times 10^{-10}} = 2.81 \times 10^9 \text{ years}$. (I left out the exponential parts in the numerator and denominator of the fraction for natural log, as they were the same and would have divided out.)

3) Here is the balanced equation: $^{210}\text{Rn} + _{\beta}\text{→} ^{210}\text{At} + \gamma$ (which doesn’t tell you much, but is a good hint.)
   We are given the energy of the gamma photon as 2.368 MeV. We are also told that the starting isotopic mass is 209.989669 amu. Electron capture does not really change the mass of the isotope much, so our value at the end should not be that very far from this starting value. From our text, we find that 1 MeV = $1.60217733 \times 10^{13}$ J, and we can
calculate the $m_e = 0.0005485798959$ amu, which we will use in the problem. Plugging the energy above into Einstein’s $E = mc^2$, which we’ve rearranged to: $m = \frac{E}{c^2}$ we get: $m = \frac{3.79396 \times 10^{-13} J}{(2.9979 \times 10^8)^2} = 4.2214 \times 10^{-30}$ kg. This value, when converted into amu, yields: $amu = (4.2214 \times 10^{-30}$ kg)/(1 amu/1.6605402 x 10^{-27} kg) = 0.00254 amu, (which is the mass difference or mass defect). The mass on the reactants side is: 209.989669. Finally, the isotopic mass is : (209.989669 - 0.002542185) = 209.9871268 amu.

4a) Probably the best answer to this part is that Ca is naturally occurring in rock, whereas Ar is not.

b) The ratio $^{40}\text{Ar}/^{40}\text{K}$ = 0.95 means that currently we have a rock with 100 parts $^{40}\text{K}$ and 95 part $^{40}\text{Ar}$ (for example). This also means that you started with 195 parts $^{40}\text{K}$. Therefore, our needed ratio for our expression is 1/1.95 = 0.5128. Given our half-life, we can get our decay constant: $k = \ln(2)/1.27 \times 10^9$ yrs = 5.46 x 10^{-10} yr^{-1}. Therefore, our age of the rock is: $t = \frac{\ln(0.5128)}{-5.46 \times 10^{-10}$ yr^{-1}} = 1.22 \times 10^9$ years

5) This problem is merely a mass-energy equivalence problem, with the added bonus of some electrochemistry. Let's do the mass-energy equivalence first.

We have 1.00 g each of hydrogen and anti-hydrogen, or 2.00 g of material. Substituting into $E = mc^2$, we get: $E = (0.002$ kg)/(2.9979 x 10^8 m/s)^2 = -1.7975 x 10^{11} kJ (which is a lot of energy). Harvesting only 33% of this energy still yields -5.932 x 10^{13} J.

The standard Zn/Cu cell has $E^\circ_{cell}$ of: 0.34-(-0.76) = 1.1 volts, which corresponds to a $\Delta G^\circ = -nFE^\circ_{cell} = (2)(96500)(1.1) = -2.123 \times 10^5 J$. To get the same energy as the nuclear cell, we would need: moles of Zn = $\frac{5.932 \times 10^{13}}{2.123 \times 10^5} = 2.794 \times 10^8$ moles. This number of moles corresponds to $1.827 \times 10^{10}$ g of Zn! (Which is QUITE a lot, and probably weighs more than the entire starship!!)