CHEM 101
Chemistry II
Problem Set IX
Answers

1) I’ve given just the solutions, without the logic spelled out. I’ve also left off the subscripts, which are the atomic numbers.
   a) $^{52}\text{Fe} \rightarrow ^{52}\text{Mn} + \beta + \gamma$
   b) $^{123}\text{Te} + \beta \rightarrow ^{123}\text{Sb}$
   c) $^{59}\text{Fe} \rightarrow ^{59}\text{Co} + \beta + \gamma$
   d) $^{212}\text{Bi} \rightarrow ^{212}\text{Po} + \beta$
   e) $^{8}\text{B} \rightarrow ^{8}\text{Be} + \beta$

2) We can assume that all the Pb came from the U. Therefore, the total amount of $^{238}\text{U}$ that was present initially is $(5.37 + 2.52)$ mg = 7.89 mg. We now have 5.37 mg of $^{238}\text{U}$.
   We can use these values for $N_t$ and $N_0$ in our equation $\ln \left( \frac{N_t}{N_0} \right) = -kt$. Given the $t_{1/2} = 4.5 \times 10^9$ years, we can get the decay constant, $k = \frac{\ln(2)}{4.5 \times 10^9 \text{ yrs}} = 1.540 \times 10^{-10} \text{ yr}^{-1}$. Now we can substitute what we know to get $t = \text{age of the ore} = \frac{\ln \left( \frac{5.37}{7.89} \right)}{-1.540 \times 10^{-10}} = 2.50 \times 10^9 \text{ years}$.

3) Here is the balanced equation: $^{210}\text{Rn} + \beta \rightarrow ^{210}\text{At} + \gamma$ (which doesn’t tell you much, but is a good hint.)
We are given the energy of the gamma photon as 2.368 MeV. We are also told that the starting isotopic mass is 209.989669 amu. Electron capture does not really change the mass of the isotope much, so our value at the end should not be that very far from this starting value. However, we need to include the mass of the electron on the reactant side to make sure our mass defect works out correctly. From our text, we find that 1 MeV = 1.60217733 \times 10^{-13} J, and we can calculate the \( m_e = 0.0005485798959 \) amu, which we will use in the problem. Plugging the energy above into Einstein’s \( E = mc^2 \), which we’ve rearranged to: \( m = \frac{E}{c^2} \) we get: \( m = \frac{3.79396 \times 10^{-13} J}{(2.9979 \times 10^8)^2} = 4.2214 \times 10^{-30} \) kg. This value, when converted into amu, yields: \( amu = (4.2214 \times 10^{-30} \) kg)(1 amu/1.6605402 \times 10^{-27} \) kg) = 0.00254 amu, (which is the mass difference or mass defect). The mass on the reactants side is: 209.989669 + 0.00054858 = 209.9902176 Finally, the isotopic mass is :(209.9902176 - 0.002542185) = 209.9876754 amu.

(An alternate treatment was presented, in which it is argued that the electron being captured, being present in the atom already, should not be included in the mass on the reactant side. If this treatment is followed, then we get the isotopic mass = (209.989669 – 0.002542185 amu) = 209.9871268 amu. This treatment feels wrong to me, although I can not put my finger on why; however, checking the definition of the amu, I find that it is correct. I therefore stand corrected.)

4a) Probably the best answer to this part is that Ca is naturally occurring in rock, whereas Ar is not.

b) The ratio \(^{40}\text{Ar}/^{40}\text{K} = 0.95\) means that currently we have a rock with 100 parts \(^{40}\text{K}\) and 95 part \(^{40}\text{Ar}\) (for example). This also means that you started with 195 parts \(^{40}\text{K}\). Therefore, our needed ratio for our expression is 1/1.95 = 0.5128. Given our half-life, we can get our decay constant: \( k = \ln(2)/1.27 \times 10^9 \) yrs = 5.46 \times 10^{10} \) yr\(^{-1}\). Therefore, our age of the rock is: \( t = \frac{\ln(0.5128)}{-5.46 \times 10^{-10} \text{yr\(^{-1}\)}} = 1.22 \times 10^9 \) years

5) This problem is merely a mass-energy equivalence problem, with the added bonus of some electrochemistry. Let’s do the mass-energy equivalence first.
We have 1.00 g each of hydrogen and anti-hydrogen, or 2.00 g of material. Substituting into $E = mc^2$, we get: $E = (0.002 \text{ kg})(2.9979 \times 10^8 \text{ m/s})^2 = 1.7975 \times 10^{11} \text{ kJ}$ (which is a lot of energy). Harvesting only 33% of this energy still yields $5.932 \times 10^{13} \text{ J}$. The standard Zn/Cu cell has $E_{\text{cell}}^\circ$ of: $0.34 - (-0.76) = 1.1 \text{ volts}$, which corresponds to a $\Delta G^\circ = nFE_{\text{cell}}^\circ = (2)(96500)(1.1) = 2.123 \times 10^5 \text{ J}$. To get the same energy as the nuclear cell, we would need: moles of Zn = $\frac{5.932 \times 10^{13}}{2.123 \times 10^5} = 2.794 \times 10^8$ moles. This number of moles corresponds to $1.827 \times 10^{10}$ g of Zn! (Which is QUITE a lot, and probably weighs more than the entire starship!!)